

**ANALYSIS OF THE DYNAMICS AND STABILITY  
OF VISCO- ELASTIC PIPES CONVEYING A  
NON-NEWTONIAN FLUID**

**BY**

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***CERTIFICATION***

This is to certify that the thesis

**“ANALYSIS OF THE DYNAMICS AND STABILITY OF VISCO- ELASTIC  
PIPES CONVEYING A NON-NEWTONIAN FLUID”**

Submitted to the School of Post-Graduate Studies University of Lagos for the award  
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is a record of original research carried out

by

**KUYE, Sidikat Ibiyemi**

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## **DEDICATION**

This work is dedicated to the Almighty God and those struggling to add values to themselves despite all odds.

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## NOTATION

### Model Problem 1: Vibration and Stability of Viscoelastic Pipe Conveying a Non-Newtonian Fluid (S. I. Units system is adopted)

$w$	-	transverse displacement (m)
$\bar{w}$	-	non-dimensionalised transverse displacement
$u$	-	longitudinal displacement (m)
$\bar{u}$	-	non-dimensionalised longitudinal displacement
$x$	-	space coordinate (m)
$\bar{x}$	-	non-dimensionalised space coordinate
$t$	-	time (s)
$\bar{t}$	-	non-dimensionalised time
$m_f$	-	mass of flowing fluid (kg/m)
$m_p$	-	mass of pipe (kg/m)
$M$	-	sum of masses of pipe and fluid (kg/m)
$v$	-	velocity of flowing fluid (m/s)
$v'$	-	differential velocity with respect to $x$ ( $s^{-1}$ )
$T_o$	-	pre-stress (tensile stress) ( $N/m^2$ )
$F_1(t)$	-	external force in the transverse direction (N)
$F_2(t)$	-	external force in the longitudinal direction (N)
$C_1$	-	damping force per unit velocity in the transverse direction
$C_2$	-	damping force per unit velocity in the axial direction
$C_D$	-	hydrodynamic drag coefficient
$p_h$	-	hydrodynamic effect of the ocean ( $N/m^2$ )
$k$	-	stiffness of the sea bed (N/m)
$\theta$	-	temperature ( $^{\circ}C$ )
$\theta'$	-	temperature gradient ( $^{\circ}C/m$ )
$\Delta\theta$	-	temperature change ( $^{\circ}C$ )
$\Delta p$	-	pressure change ( $N/m^2$ )
$pA$	-	pressurization effect (N)
$p_o$	-	pressure at entry ( $N/m^2$ )

$E$	-	Young modulus (N/m <sup>2</sup> )
$A$	-	pipe cross sectional area after deformation (m <sup>2</sup> )
$A_o$	-	original cross sectional area of pipe (m <sup>2</sup> )
$A'$	-	change in the surface area of the pipe (m <sup>2</sup> )
$D_1$	-	inner diameter of pipe (m)
$D_2$	-	outer diameter of pipe (m)
$L$	-	length of pipe (m)
$\gamma$	-	coefficient of area deformation
$\alpha$	-	coefficient of thermal expansion (°C <sup>-1</sup> )
$g$	-	acceleration due to gravity (m/s <sup>2</sup> )
$z$	-	depth of the pipe below the sea level (h)
$r$	-	changing radius of the pipe (m)
$R$	-	nominal radius of the pipe (m)
$\tilde{w}$	-	non-dimensionalised transverse response in Laplace plane
$\bar{w}^F$	-	non-dimensionalised transverse response in Fourier plane
$\tilde{w}^F$	-	non-dimensionalised transverse response in Fourier-Laplace plane
$\rho_w$	-	density of water (kg/m <sup>3</sup> )
$I$	-	pipe moment of inertia (m <sup>4</sup> )
$p_h$	-	hydrodynamic effect of the ocean (N/m <sup>2</sup> )
$\eta$	-	viscoelastic constant

## **Model Problem 2: Vibration and Stability of Sandwich Viscoelastic Pipes**

### **Conveying a Non-Newtonian Fluid (S. I. Units system is adopted)**

$w$	-	transverse displacement (m)
$\bar{w}$	-	non-dimensionalised transverse displacement
$u$	-	longitudinal displacement (m)
$\bar{u}$	-	non-dimensionalised longitudinal displacement
$x$	-	space coordinate (m)
$\bar{x}$	-	non-dimensionalised space coordinate
$t$	-	time (s)
$\bar{t}$	-	non-dimensionalised time

$M$	-	sum of masses of pipe and fluid (kg/m)
$m_f$	-	mass of flowing fluid (kg/m)
$v$	-	velocity of flowing fluid (m/s)
$v'$	-	differential velocity with respect to $x$ ( $s^{-1}$ )
$D_1$	-	inner diameter of inner pipe (m)
$D_2$	-	outer diameter of inner pipe (m)
$D_2$	-	inner diameter of outer pipe (m)
$D_3$	-	outer diameter of outer pipe (m)
$T_o$	-	pre-stress (tensile stress) ( $N/m^2$ )
$F_1(t)$	-	external force in the transverse direction (N)
$F_2(t)$	-	external force in the longitudinal direction (N)
$C_1$	-	damping force per unit velocity in the transverse direction
$C_2$	-	damping force per unit velocity in the axial direction
$C_D$	-	hydrodynamic drag coefficient
$I$	-	moment of inertia ( $m^4$ )
$p_h$	-	hydrodynamic effect of the ocean ( $N/m^2$ )
$k$	-	stiffness of the sea bed (N/m)
$\theta$	-	temperature ( $^{\circ}C$ )
$\theta'$	-	temperature gradient ( $^{\circ}C/m$ )
$\Delta\theta$	-	temperature change ( $^{\circ}C$ )
$\Delta p$	-	pressure change ( $N/m^2$ )
$pA_1$	-	pressurization effect (N)
$p_o$	-	pressure at entry ( $N/m^2$ )
$E$	-	Young modulus ( $N/m^2$ )
$A_1$	-	inner pipe cross sectional area after deformation ( $m^2$ )
$A_2$	-	outer pipe cross sectional area after deformation ( $m^2$ )
$A_{o1}$	-	inner pipe original cross sectional area ( $m^2$ )
$A_{o2}$	-	outer pipe original cross sectional area of pipe ( $m^2$ )
$A'$	-	change in the surface area of the pipe ( $m^2$ )

$L$	-	length of pipe (m)
$\gamma$	-	coefficient of area deformation
$\alpha$	-	coefficient of thermal expansion ( $^{\circ}\text{C}^{-1}$ )
$g$	-	acceleration due to gravity ( $\text{m/s}^2$ )
$z$	-	depth of the pipe below the sea level (m)
$r$	-	changing radius of the pipe (m)
$R$	-	nominal radius of the pipe (m)
$\tilde{W}$	-	non-dimensionalised transverse response in Laplace plane
$\overline{W}^F$	-	non-dimensionalised transverse response in Fourier plane
$\tilde{\overline{W}}^F$	-	non-dimensionalised transverse response in Fourier-Laplace plane
$\rho_w$	-	density of water ( $\text{kg/m}^3$ )
$m_{p1}$	-	mass of inner pipe (kg/m)
$m_{p2}$	-	mass of outer pipe (kg/m)
$F_1(t)$	-	external force in the transverse direction (N)
$F_2(t)$	-	external force in the longitudinal direction (N)
$C_1$	-	damping force per velocity in the transverse direction
$C_2$	-	damping force per velocity in the axial direction
$C_D$	-	hydrodynamic drag coefficient
$I_1$	-	inner pipe moment of inertia ( $\text{m}^4$ )
$I_2$	-	outer pipe moment of inertia ( $\text{m}^4$ )
$p_h$	-	hydrodynamic effect of the ocean ( $\text{N/m}^2$ )
$E_1$	-	Young modulus of inner pipe ( $\text{N/m}^2$ )
$E_2$	-	Young modulus of outer pipe ( $\text{N/m}^2$ )
$\mu_1$	-	inner pipe frictional coefficient
$\mu_2$	-	outer pipe frictional coefficient
$\alpha_1$	-	inner pipe coefficient of thermal expansion ( $^{\circ}\text{C}^{-1}$ )
$\alpha_2$	-	outer pipe coefficient of thermal expansion ( $^{\circ}\text{C}^{-1}$ )
$\eta_1$	-	viscoelastic constant of inner pipe
$\eta_2$	-	viscoelastic constant of outer pipe

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## ABSTRACT

*Flow induced vibration of pipeline and riser systems are strongly dependent on internal fluid flow parameters as well as the mechanical properties of the conveyance vessel. The steady growing demand on the transport of primary energy sources such as oil and gas necessitated larger diameters and higher operating pressures. In order to reduce the amount of steel in the pipes there was a search for higher strength materials that could be used to reduce the wall thickness, as permitted by advances in manufacturing processes. Some studies on the mechanics of sandwich elastic systems as effective vibration and noise reduction mechanisms have stimulated the possibility of replacing stainless steel pipes with sandwich pipes especially in deepwater environment. Recent analysis has indicated that crude oil is a Newtonian fluid at high temperature, but exhibits non-Newtonian behaviour due to its complex mixture of hydrocarbons.*

*This thesis presents our attempt to find an alternative material to steel as offshore fluid conveyance medium, by analytically investigating the dynamics of viscoelastic pipes in offshore environment using Euler-Bernoulli beam theory. By idealising the viscoelastic pipeline resting on the sea bed as a viscoelastic beam that is resting on an elastic continuum, a non-linear, boundary value partial differential equation governing the fluid- structure- soil interaction mechanics is formulated. The material property of the beam-model pipe is described by the Kelvin-Voigt type viscoelastic constitutive relation. By linearizing the governing partial differential equation matching the problem physics, under slight perturbation of the internal fluid velocity and other flow variables closed form analytical results for the system dual natural frequencies and consequently stability under external excitation are computed for field designs.*

*We were able to find out that stability for both single and sandwich viscoelastic pipes is low in offshore environment compared with that of steel pipe. However, when sandwich pipes are arranged in such a way that thicker viscoelastic material is on the inside and thinner steel material on the outside, natural frequency is tremendously improved which demonstrates good stability. Results also show that the more the flow behaviour index of the conveyed fluid the more the stability, which makes dilatant fluid to be more stable than Newtonian fluid which in turn is more stable than pseudoplastic fluid. In the case of sandwiched pipes, the results obtained in this work emphasized the importance of arrangement of the pipes for offshore applications.*