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Charles Osheku
National Space Research and Development Ag…


Theddeus Akano
University of Lagos

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Analysis of Flow Induced Acoustic Waves in a Vibrating Offshore Pipeline

Charles A. Osheku
Department of Systems Engineering
Faculty of Engineering
University of Lagos
Lagos, Nigeria

Vincent O.S Olunloyo
Department of Systems Engineering
Faculty of Engineering
University of Lagos
Lagos, Nigeria

Theddeus T. Akano
Department of Systems Engineering
Faculty of Engineering
University of Lagos
Lagos, Nigeria

Abstract

Flow induced acoustic wave characteristics in a vibrating subsea pipeline is investigated. For this problem, acoustic wave equations are formulated and matched for the various vibrating segments. The pipeline system is idealized as a network of fluid conveyance elastic beams resting on a moving seabed via recent advances in subsea pipeline vibrations. By employing operational methods, closed forms results as influenced by internal fluid variables and subsea soil geotechnical properties, are computed for design applications. It is further shown that the vibration of any pipeline system is modulated by transverse, longitudinal and vibro-acoustic natural frequencies.

Key words; Acoustic waves, subsea pipeline, fluid and subsoil properties

1.0 Introduction

The study of acoustic pressure waves propagation from industrial plants, aircraft engines and noise generating machines has remain an active research area for several decades. However, limited literatures exist in the area of acoustic- structure dynamic interaction modeling. Nevertheless, within the context of analytical and experimental studies in acoustic – structure dynamics, a number of investigations have been reported in refs.[1-10].
For these problems, analytical techniques were employed to study active control of acoustic-structure interaction in 2-D and 3-D enclosures. In particular, Pota [5], investigated acoustic-structure interaction through a 3-D rectangular enclosure to improve control design analysis.
In the field of acoustic-structure interaction, the scarcity of literature in analytical modeling of pressure waves propagation, through geometrical enclosures having complex boundary conditions has been attributed partly to the intractability of dynamic equations which preclude closed form solutions. For some cases, with simplified assumptions, closed-form solutions are possible usually with rigorous mathematical intrigues and manipulations.
However, for 3-D acoustic enclosures, with a vibrating boundary surfaces, these identification techniques are fronted with difficulty, especially with two-input one-output systems. Nevertheless, investigations of vibro-acoustic pressure waves propagation through an offshore pipeline system under the influence of ocean environmental forces and flow variables are not widely reported in literature.

Within the context of acoustic–structure interaction through pipes, a number of interesting results have been reported in refs.[11-17]. Flow induced acoustic wave characteristics in a vibrating subsea pipeline is investigated. For this problem, a generalized partial differential equation governing the vibro-acoustic pressure waves propagation through a pipeline system is formulated and matched for each segment of the pipeline. This paper is organized as follows. Section 1 introduces the problem under investigation within a general context. In the next section, the essential analytical mechanics is briefly reviewed. In section 3, these relationships are incorporated into a simplified analytical model for the mathematical analysis of the flow induced- acoustic pressure waves problem. In section 4, closed forms results for the vibro-acoustic pressure waves, transverse, longitudinal and vibro-acoustic natural frequencies of pipeline system are reported whilst in section 5, the paper ends with the summary and conclusion.

2.0 Problem Formulation

The mathematical problem of interest is to examine analytically, the propagation of flow induced acoustic pressure wave through an offshore pipeline system idealized a fluid conveyance elastic beam as illustrated in Figure 1.

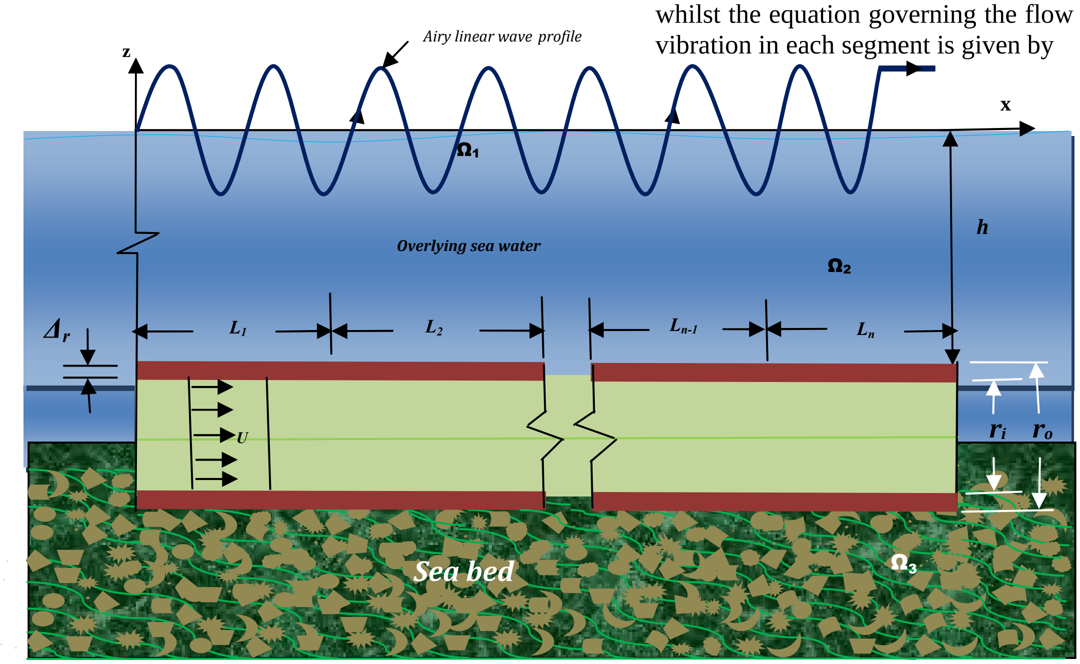

Figure 1: Flow Geometry on Homogeneous Seabed Subsoil Layer.

The generalized flow induced acoustic wave equation is given by the relation.

$$\nabla^2 P - \frac{1}{c^2}\frac{\partial^2 P}{\partial t^2} - \frac{2U}{c^2}\frac{\partial^2 P}{\partial t \partial x} - \frac{U^2}{c^2}\frac{\partial^2 P}{\partial x^2} = -\frac{1}{\rho}\frac{\partial^2 W}{\partial t^2} \qquad (1)$$

where: $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial x^2}$

whereas the linearized equation governing the transverse vibration of each segment, following Olunloyo et al [20], satisfy the form viz

$$mW + \left(C_1 + C_D\right)W + 2m_f U\dot{W}' - \left(T_o - pA - m_f U^2 - \varepsilon_t EA_t\overline{\Theta}\right)W''$$
$$-\left(\frac{\Delta p}{L}A\right)W' - mg + EIW^{IV} + k_b W = P_h(t)A_p \qquad (2)$$

whilst the equation governing the flow induced longitudinal vibration in each segment is given by

$$m\ddot{u} + C_2\dot{u} + 2m_f U\dot{u}' + p'A + pA' + \alpha EA_t\Theta' + \alpha EA_t{}'\Theta$$
$$+(T_o - EA_t - pA - \alpha EA_t\Theta)w'w'' - (EA_t - m_f U^2)u'' - \left(p'A + pA' + \alpha EA_t{}'\Theta + \alpha EA_t\Theta'\right)\frac{w'^2}{2} \qquad (3)$$
$$-EA_t'(u' + \frac{1}{2}w'^2) - EI(w^{IV}w' + w''w''') + C_D\dot{u} + \mu mg = 0$$

On introducing the following dimensionless parameters

$$\bar{x}=\frac{x}{L}\quad \bar{w}=\frac{w}{L},\quad \bar{u}=\frac{u}{L},\quad \delta_1=\frac{m_f}{m},\quad \delta_2=\frac{m_w}{M}\quad \tau=L^2\sqrt{\frac{m}{EI}}\quad \bar{t}=\sqrt{\frac{EI}{m}}\frac{t}{L^2},\ \bar{U}=UL\sqrt{\frac{m_f}{EI}}\ ;$$
$$\bar{A}=\frac{A}{L^2}\quad \beta_o=\frac{T_oL^2}{EI},\quad \bar{p}\bar{A}=\frac{pA_oL^2}{EI},\quad \bar{C}_1=\frac{c_1L^2}{\sqrt{mEI}},\quad \bar{C}_2=\frac{c_2L^2}{\sqrt{mEI}}\ ;\ \bar{C}_D=\frac{c_DL^2}{\sqrt{mEI}}\quad \beta_1=\frac{EA_oL^2}{EI}\ ; \tag{4}$$
$$\bar{g}=\frac{mgL^3}{EI}\quad \bar{k}_b=\frac{k_bL^4}{EI},\quad \bar{P}_h(\bar{t})\bar{A}_p=\frac{P_h(t)A_pL^3}{EL};\ \bar{r}=\frac{r}{L}\quad \bar{c}=cL\sqrt{\frac{m}{EI}}\quad \bar{\theta}=\frac{\theta}{\theta_0}\quad ;\quad \varepsilon_t=\alpha\theta_0$$

Eqs.(2-3) can be non-dimensionalized into the forms

$$\frac{\partial^4\bar{W}}{\partial\bar{x}^4}+\left(\left(2\sqrt{\delta_1}+1\right)\bar{U}^2-\beta_0+\bar{p}\bar{A}+\varepsilon_t\beta_1\left(1-\frac{\gamma}{2}\right)\bar{\Theta}\right)\frac{\partial^2\bar{W}}{\partial\bar{x}^2}+\frac{\partial^2\bar{W}}{\partial\bar{t}^2}+\left(\bar{C}_1-\frac{\Delta\bar{p}\bar{A}}{\bar{U}}\left(1-\frac{\gamma}{2}\right)\right)\frac{\partial\bar{W}}{\partial\bar{t}}-\bar{g}+k_b\bar{W}=\bar{P}_h(\bar{t})\bar{A}_p \tag{5}$$

Subject to the pinned-pinned end boundary conditions namely

$$\bar{W}(0,\bar{t})=\bar{W}(1,\bar{t})=\bar{W}_{xx}(0,\bar{t})=\bar{W}_{xx}(1,\bar{t})=0 \tag{6}$$

In the meantime, the non-dimensional equations governing the interaction of the two domains viz; $\Omega_1\cup\Omega_2$ satisfy the forms

$$\nabla^2\bar{\Phi}=0$$
$$\frac{\bar{P}_h}{\bar{\rho}_w}+\frac{\partial\bar{\Phi}}{\partial\bar{t}}+\bar{g}\bar{z}+\frac{1}{2}\left|\nabla\bar{\Phi}\right|^2=0 \tag{8}$$

where the following defined

$$\bar{z}=\frac{z}{L};\qquad M=m+m_w;\qquad \bar{\Phi}=\sqrt{\frac{M}{EI}}\Phi \tag{9}$$

The first part of Eqn.(8) admits a solution of the form $\bar{\Phi}=\bar{\phi}(\bar{x},\bar{y})\bar{f}(\bar{z},\bar{t})$ so that on introducing variables separation Eqn.(8) now takes the form

and the relation

$$\nabla^2\bar{\phi}\ \bar{f}(\bar{z},\bar{t})+\bar{\phi}\frac{\partial^2\bar{f}(\bar{z},\bar{t})}{\partial\bar{z}^2}=0 \tag{10a}$$

or as

$$\frac{\nabla^2\bar{\phi}(\bar{x},\bar{y})}{\bar{\phi}(\bar{x},\bar{y})}=-\frac{\partial^2\bar{f}(\bar{z},\bar{t})}{\partial\bar{z}^2}\Big/\bar{f}(\bar{z},\bar{t})=-k^2 \tag{10b}$$

Further simplification leads to the well known Helmholtz equation for $\bar{\phi}$ namely $\bar{\nabla}^2\bar{\phi}+k^2\bar{\phi}=0$ while $\bar{f}(\bar{z},\bar{t})$ is governed by

$$\frac{\partial^2\bar{f}(\bar{z},\bar{t})}{\partial\bar{z}^2}-k^2\bar{f}(\bar{z},\bar{t})=0 \tag{11}$$

subject to the free surface boundary condition

$$\frac{\partial\bar{f}(\bar{z},\bar{t})}{\partial\bar{z}}=-\frac{1}{g}\frac{\partial\bar{f}(\bar{z},\bar{t})}{\partial\bar{z}}\ \text{at}\ \bar{z}=0 \tag{12a}$$

$$\frac{\partial \bar{f}\left(\bar{z},\bar{t}\right)}{\partial \bar{z}} = \frac{\partial \overline{w}}{\partial \bar{t}} \quad \forall \quad \bar{z} = -\bar{h} \tag{12b}$$

In the meantime, Eq.(5) can be rewritten as

$$\frac{\partial^4 \overline{W}}{\partial \bar{x}^4} + \left((2\sqrt{\delta_1}+1)\overline{U}^2 - \beta_o + \bar{p}\overline{A}_o(1-\frac{\gamma}{2}) + \varepsilon_t \beta_1 (1-\frac{\gamma}{2})\overline{\Theta} \right)\frac{\partial^2 \overline{W}}{\partial \bar{x}^2} + \frac{\partial^2 \overline{W}}{\partial \bar{t}^2} +$$
$$\left(\overline{C}_1 + \overline{C}_D - \left(\frac{\Delta\bar{p}\overline{A}_o(1-\frac{\gamma}{2}) + \bar{p}\overline{A}_o\gamma + \varepsilon_t\beta_1\Delta\overline{\Theta}(1-\frac{\gamma}{2}) + \varepsilon_t\beta_1\overline{\Theta}\gamma}{\overline{U}} \right)\right)\frac{\partial \overline{W}}{\partial \bar{t}} - \bar{g} + \bar{k}_b\overline{W} \tag{13}$$
$$= 2\frac{\delta_2}{\bar{r}_o}\left(\frac{\partial \overline{\Phi}}{\partial \bar{t}} + \frac{\bar{g}\bar{z}}{1-\delta_2} \right)$$

By introducing the Laplace integral transform viz:

$$\left(\tilde{\cdot}\right) = \int_0^\infty (\cdot) e^{-st} dt, (\cdot) = \frac{1}{2\pi i}\int_{\eta - i\infty}^{\eta + i\infty} \left(\tilde{\cdot}\right) e^{st} ds \tag{14}$$

we can rewrite Eq. (13) in the Laplace transform plane as :

$$\frac{d^4 \overline{W}(\bar{x},s)}{d\bar{x}^4} + \left(s^2 \tilde{\overline{W}}(\bar{x},s) - s\tilde{\overline{W}}(\bar{x},0) - \dot{\overline{W}}(\bar{x},0)\right) + \left((2\sqrt{\delta_1}+1)\overline{U}^2 - \beta_o + \bar{p}\overline{A}_o(1-\frac{\gamma}{2}) + \varepsilon_t\beta_1(1-\frac{\gamma}{2})\overline{\Theta}\right)\frac{d^2 \tilde{\overline{W}}(\bar{x},s)}{d\bar{x}^2}$$
$$+ \overline{K}_b\tilde{W}(x,s)\left(\overline{C}_1 + \overline{C}_D - \left(\frac{\Delta\bar{p}\overline{A}_o(1-\frac{\gamma}{2}) + \bar{p}\overline{A}_o\gamma + \varepsilon_t\beta_1\Delta\overline{\Theta}(1-\frac{\gamma}{2}) + \varepsilon_t\beta_1\overline{\Theta}\gamma}{\overline{U}} \right)\right)\left(s\tilde{\overline{W}}(\bar{x},s) - W(\bar{x},0)\right) - \frac{\bar{g}}{s}$$
$$= 2\frac{\delta_2}{\bar{r}_o}\left(\left(s\tilde{\overline{\Phi}}(\bar{x},s,\bar{z}) + \frac{\bar{g}\bar{z}}{s(1-\delta_2)}\right)\right) \quad \forall \quad \bar{z} = -\bar{h} \tag{15}$$

On further introduction of Fourier Finite Sine transform namely

$$\left[\tilde{\cdot}\right]^F = \int_0^1 \left[\tilde{\cdot}\right] \sin n\pi\bar{x} d\bar{x}, \quad \left[\tilde{\cdot}\right] = 2\sum_{n=1}^{\infty}\left[\tilde{\cdot}\right]^F \sin n\pi\bar{x} \tag{16}$$

subject to the following boundary conditions at $\bar{x} = (0,1)$ namely

$$\tilde{\overline{W}}(o,s) = \tilde{\overline{W}}(1,s) = \frac{d^2\tilde{\overline{W}}(0,s)}{d\bar{x}^2} = \frac{d^2\tilde{\overline{W}}(1,s)}{d\bar{x}^2} = 0 \tag{17}$$

we can rewrite Eq. (15) by assuming zero initial conditions as:

$$n^4\pi^4\tilde{\tilde{W}}\left(\bar{\lambda}_n,s\right)^F+s^2\tilde{\tilde{W}}\left(\bar{x},s\right)+\left((2\sqrt{\delta_1}+1)\bar{U}^2-\beta_o+\bar{p}\bar{A}_o(1-\frac{\gamma}{2})+\varepsilon_t\beta_1(1-\frac{\gamma}{2})\bar{\Theta}\right)n^2\pi^2\tilde{\tilde{W}}\left(\bar{\lambda}_n,s\right)^F$$
$$+\bar{K}_b\tilde{\tilde{W}}\left(\bar{\lambda}_n,s\right)^F+\left(\bar{C}_1+\bar{C}_D-\left(\frac{\Delta\bar{p}\bar{A}_o(1-\frac{\gamma}{2})+\bar{p}\bar{A}_o\gamma+\varepsilon_t\beta_1\Delta\bar{\Theta}(1-\frac{\gamma}{2})+\varepsilon_t\beta_1\bar{\Theta}\gamma}{\bar{U}}\right)\right)s\tilde{\tilde{W}}\left(\bar{\lambda}_n,s\right)^F-\frac{\bar{g}}{s} \tag{18}$$
$$=2\frac{\delta_2}{\bar{r}_o}\left(\left(s\tilde{\tilde{\Phi}}^F\left(\bar{\lambda}_n,s,\bar{z}\right)\Big|_{z=-\bar{h}}+\frac{\bar{g}\bar{h}}{s(1-\delta_2)}\bar{1}^F\right)\right\}\right) \quad \forall \quad \bar{z}=-\bar{h}$$

where $\bar{1}^F=\int_0^1\sin n\pi\bar{x}d\bar{x}=\left(\frac{\left(1+(-1)^{n+1}\right)}{n\pi}\right)$

Following Olunloyo et al [21], we find, on applying Eqs. (12a-b), that

$$\tilde{\tilde{\Phi}}\left(\bar{\lambda}_n,s,\bar{z}\right)^F=\phi\left(\bar{\lambda}_n\right)^F\tilde{\tilde{F}}\left(\bar{\lambda}_n\right)=-\bar{u}_W\frac{s}{\bar{k}}\tilde{\tilde{W}}\left(\bar{x},s\right)\bar{1}^F\frac{\cosh\bar{k}\bar{z}}{\sinh\bar{k}\bar{h}}=\hat{\beta}s\tilde{\tilde{W}}\left(\bar{\lambda}_n,s\right)^F \tag{19}$$

Use of Eq, (19) allow us to rewrite Eq. (18) in the form

$$n^4\pi^4\tilde{\tilde{W}}\left(\bar{\lambda}_n,s\right)^F+s^2\tilde{\tilde{W}}\left(\bar{x},s\right)+\left((2\sqrt{\delta_1}+1)\bar{U}^2-\beta_o+\bar{p}\bar{A}_o(1-\frac{\gamma}{2})+\varepsilon_t\beta_1(1-\frac{\gamma}{2})\bar{\Theta}\right)n^2\pi^2\tilde{\tilde{W}}\left(\bar{\lambda}_n,s\right)^F$$
$$+\bar{K}_b\tilde{\tilde{W}}\left(\bar{\lambda}_n,s\right)^F+\left(\bar{C}_1+\bar{C}_D-\left(\frac{\Delta\bar{p}\bar{A}_o(1-\frac{\gamma}{2})+\bar{p}\bar{A}_o\gamma+\varepsilon_t\beta_1\Delta\bar{\Theta}(1-\frac{\gamma}{2})+\varepsilon_t\beta_1\bar{\Theta}\gamma}{\bar{U}}\right)\right)s\tilde{\tilde{W}}\left(\bar{\lambda}_n,s\right)^F-\frac{\bar{g}}{s} \tag{20}$$
$$=2\frac{\delta_2}{\bar{r}_o}\left(\left(s^2\bar{\beta}\tilde{\tilde{W}}\left(\bar{\lambda}_n,s\right)-\frac{\bar{g}\bar{h}}{s(1-\delta_2)}\bar{1}^F\right)\right\}\right) \quad \forall \quad \bar{z}=-\bar{h}$$

where $\hat{\beta}=-\frac{\bar{u}_W}{\bar{k}\sinh\bar{k}\bar{h}}\cosh\bar{h}\bar{k}\bar{z} \quad ; \quad \bar{\beta}=\hat{\beta}\Big|_{z=-\bar{h}}=\frac{-\bar{u}_W}{\bar{k}}\coth\bar{k}\bar{h}$ (21)

We can then solve Eq. (20) for $\tilde{\tilde{W}}$ as

$$\tilde{\tilde{W}}\left(\lambda_n,s\right)=\left\{\frac{\left(1-\frac{2\bar{h}}{(1-\delta_2)}\frac{\delta_2}{\bar{r}_o}\right)\bar{1}^F\bar{g}}{\left(1-2\bar{\beta}\frac{\delta_2}{\bar{r}_o}\right)s\left(s^2+s\bar{\eta}_1+\bar{\eta}^2\right)}\right\} \tag{22}$$

Where: $$\overline{\eta}_1 = \left(\frac{\left(\overline{C}_1 + \overline{C}_D - \left(\frac{\Delta\overline{p}\overline{A}_o(1-\frac{\gamma}{2}) + \overline{p}\overline{A}_o\gamma + \varepsilon_t\beta_1\Delta\overline{\Theta}(1-\frac{\gamma}{2}) + \varepsilon_t\beta_1\overline{\Theta}\gamma}{\overline{U}} \right) \right)}{\left(1 - 2\overline{\beta}\,\frac{\delta_2}{\overline{r}_o}\right)} \right) \quad (23a)$$

$$\overline{\eta}^2 = \left(\frac{n^4\pi^4 - \left((2\sqrt{\delta_1}+1)\overline{U}^2 - \beta_o + \overline{p}\overline{A}_o(1-\frac{\gamma}{2}) + \varepsilon_t\beta_1(1-\frac{\gamma}{2})\overline{\Theta} \right) n^2\pi^2 + \overline{K}_b}{\left(1 - 2\overline{\beta}\,\frac{\delta_2}{\overline{r}_o}\right)} \right) \quad (23b)$$

By substituting $s = \pm i\omega_n$ into the characteristic equation $\left(s^2 + s\overline{\eta}_1 + \overline{\eta}^2\right)$ leads to the two complimentary natural frequencies

$$\overline{\omega}_{n(1)}^{\;2} = \overline{\eta}^2 - \frac{\overline{\eta}_1^{\;2}}{2} \;;\quad \overline{\omega}_{n(2)}^{\;2} = -\overline{\eta}^2 \quad (24)$$

whilst the non-dimenensionalized form of Eq.(3) takes the form;

$$\begin{aligned}
&\frac{\partial^2\overline{u}}{\partial\overline{t}^2} + (\overline{C}_2 + \overline{C}_D)\frac{\partial\overline{u}}{\partial\overline{t}} + \left(\left(2\sqrt{\delta_1}+1\right)\overline{U}^2 - \beta_1\left(1-\gamma/2\right) \right)\frac{\partial^2\overline{u}}{\partial\overline{x}^2} \\
&+ \left(\beta_O - \beta_1\left(1-\gamma/2\right) - \overline{p}\overline{A}_o\left(1-\gamma/2\right) - \varepsilon_t\beta_1\left(1-\gamma/2\right)\overline{\Theta} \right)\frac{\partial\overline{w}}{\partial\overline{x}}\frac{\partial^2\overline{w}}{\partial\overline{x}^2} \\
&- \left(\Delta\overline{p}\overline{A}_o\left(1-\gamma/2\right) + \overline{p}\overline{A}_o\gamma + \varepsilon_t\beta_1\left(1-\gamma/2\right)\Delta\overline{\Theta} + \varepsilon_t\beta_1\gamma\overline{\Theta} \right) \\
&+ \frac{1}{2}\left(\Delta\overline{p}\overline{A}_o\left(1-\gamma/2\right) + \overline{p}\overline{A}_o\gamma + \varepsilon_t\beta_1\left(1-\gamma/2\right)\Delta\overline{\Theta} + \varepsilon_t\beta_1\gamma\overline{\Theta} \right)\left(\frac{\partial\overline{w}}{\partial\overline{x}}\right)^2 \\
&+ \beta_1\gamma\left(\frac{\partial\overline{u}}{\partial\overline{x}} + \frac{1}{2}\left(\frac{\partial\overline{w}}{\partial\overline{x}}\right)^2 \right) - \left(\frac{\partial^4\overline{w}}{\partial\overline{x}^4}\frac{\partial\overline{w}}{\partial\overline{x}} + \frac{\partial^2\overline{w}}{\partial\overline{x}^2}\frac{\partial^3\overline{w}}{\partial\overline{x}^3} \right) = -\mu\overline{g}
\end{aligned} \quad (25)$$

We can isolate the characteristic equation from the forgoing as :

$$s^2 + \left(\overline{C}_2 + \overline{C}_D + \frac{\beta_1\gamma}{\overline{U}} \right)s - \left(\left(2\sqrt{\delta_1}+1\right)\overline{U}^2 - \beta_1\left(1-\gamma/2\right) \right)n^2\pi^2 = 0 \quad (26)$$

so that on setting $s = -i\Omega$ the longitudinal natural frequency can be computed as

$$\bar{\Omega}^2 = \left(\frac{-\left(\bar{C}_2 + \bar{C}_D + \frac{\beta_1 \gamma}{\bar{U}} \right)}{2} \right)^2 \pm \left(\left(\frac{-\left(\bar{C}_2 + \bar{C}_D + \frac{\beta_1 \gamma}{\bar{U}} \right)}{2} \right)^2 + \left(\left(2\sqrt{\delta_1} + 1 \right) \bar{U}^2 - \beta_1 \left(1 - \gamma/2 \right) \right) n^2 \pi^2 \right) \tag{27}$$

Equation (27) can be decomposed to obtain the following frequency relations:

$$\begin{aligned} \bar{\Omega}_1^2 &= \frac{\left(-\left(\bar{C}_2 + \bar{C}_D + \frac{\beta_1 \gamma}{\bar{U}} \right) \right)^2}{2} + \left(\left(2\sqrt{\delta_1} + 1 \right) \bar{U}^2 - \beta_1 \left(1 - \gamma/2 \right) \right) n^2 \pi^2 ; \\ \bar{\Omega}_2^2 &= -\left(\left(2\sqrt{\delta_1} + 1 \right) \bar{U}^2 - \beta_1 \left(1 - \gamma/2 \right) \right) n^2 \pi^2 \end{aligned} \tag{28}$$

3.0 Mathematical Analysis of the Flow Induced Acoustic Waves Equation

We shall restrict our analysis to the case of uniform flow conditions through each segment of the pipeline subject the geotechnical conditions illustrated in Figure 1

$$\bar{\nabla}^2 \bar{P} - \frac{1}{\bar{c}^2} \frac{\partial^2 \bar{P}}{\partial \bar{t}^2} - \frac{2\bar{U}}{\bar{c}^2 \sqrt{\delta_1}} \frac{\partial^2 \bar{P}}{\partial \bar{t} \partial \bar{x}} - \frac{\bar{U}^2}{\bar{c}^2 \delta_1} \frac{\partial^2 \bar{P}}{\partial \bar{x}^2} = -\frac{1}{\bar{\rho}} \frac{\partial^2 \bar{W}}{\partial \bar{t}^2}; \quad \forall \quad \bar{\nabla}^2 = \left(\frac{\partial^2}{\partial \bar{r}^2} + \frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} + \frac{1}{\theta_0^2 \bar{r}^2} \frac{\partial^2}{\partial \bar{\theta}^2} + \frac{\partial^2}{\partial \bar{x}^2} \right) \tag{29}$$

So that with the introduction of Mach number, the forgoing can be rewritten as

$$\frac{\partial^2 \bar{P}}{\partial \bar{r}^2} + \frac{1}{\bar{r}} \frac{\partial \bar{P}}{\partial \bar{r}} + \frac{1}{\theta_0^2 \bar{r}^2} \frac{\partial^2 \bar{P}}{\partial \bar{\theta}^2} + \left(1 - \frac{3M^2}{\bar{c}_0^{\,2} \delta_1} \right) \frac{\partial^2 \bar{P}}{\partial \bar{x}^2} - \frac{1}{\bar{c}_0^{\,2}} \frac{\partial^2 \bar{P}}{\partial \bar{t}^2} = -\frac{1}{\bar{\rho}} \frac{\partial^2 \bar{W}}{\partial \bar{t}^2}; \quad \forall \quad M = \frac{U}{c} \tag{30}$$

Now, on the assumption of axisymmetric flow through the segment of the pipe, Eq.(31) reduces to the form

$$\frac{\partial^2 \bar{P}_0}{\partial \bar{r}^2} + \frac{1}{\bar{r}} \frac{\partial \bar{P}_0}{\partial \bar{r}} + \left(1 - \frac{3M^2}{\bar{c}_0^{\,2} \delta_1} \right) \frac{\partial^2 \bar{P}_0}{\partial \bar{x}^2} - \frac{1}{\bar{c}_0^{\,2}} \frac{\partial^2 \bar{P}_0}{\partial \bar{t}^2} = -\frac{1}{\bar{\rho}} \frac{\partial^2 \bar{W}}{\partial \bar{t}^2}; \tag{31}$$

By introducing Finite Hankel Transform namely

$$\left. F^H \{[\bullet]\} = [\bar{\bullet}] = \int_0^{\bar{r}_0} \bar{r} ([\bullet]) R_m(\bar{r}) d\bar{r} \quad ; [\bullet] = \sum_{m=1}^{\infty} [\bar{\bullet}] R_m(\bar{r}) \ \forall \quad R_m(\bar{r}) = \frac{\sqrt{2} J_0(\bar{\lambda}_m \bar{r})}{\bar{r}_0 \bar{J}_1(\bar{\lambda}_m \bar{r}_0)} \right\} \tag{32}$$

Subject to the following Sturm-Liouville system

$$(\bar{r}R')' + \bar{\lambda}^2 rR = 0 \, , \, 0 < \bar{r} < \bar{r}_0 \quad ; R(\bar{r}_0) = 0 \tag{33}$$

in conjunction with Eq.(14).;

Eq.(33) in the transform plane, subject to zero initial conditions takes the following form viz;

$$\begin{pmatrix} \left(1-\frac{3M^2}{\bar{c}_0{}^2\delta_1}\right)\left(-n^2\pi^2\tilde{\bar{P}}_0^{FH}\left(\bar{\lambda}_n,\bar{\lambda}_m,s\right)+n\pi(-1)^{n+1}\tilde{\bar{P}}_o^{H}\left(1,\lambda_m,s\right)+n\pi\tilde{\bar{P}}_o^{H}\left(0,\bar{\lambda}_m,s\right)\right) \\ +\left(\lambda^2{}_m-\frac{s^2}{c_0{}^2}\right)\tilde{\bar{P}}_0^{FH}\left(\bar{\lambda}_n,\bar{\lambda}_m,s\right)=\frac{1}{\bar{\rho}}s^2\overline{W}^{FH}\left(\bar{\lambda}_n,\bar{\lambda}_m,s\right) \end{pmatrix} \tag{34}$$

To enable us complete the solution of Eq.(34), we shall substitute for $\overline{W}^{FH}\left(\bar{\lambda}_n,\lambda_m,s\right)$ from Eq.(22) to rewrite the forgoing as

$$\tilde{\bar{P}}_0^{FH}\left(\bar{\lambda}_n,\bar{\lambda}_m,s\right)=c_0{}^{-2}\begin{pmatrix} \dfrac{n\pi(-1)^{n+1}\tilde{\bar{P}}_o^{H}\left(1,\bar{\lambda}_m,s\right)+n\pi\tilde{\bar{P}}_o^{H}\left(0,\bar{\lambda}_m,s\right)}{\left(s^2+\Omega_1{}^2\right)} \\ +\dfrac{\left(1-\dfrac{2\bar{h}}{(1-\delta_2)}\dfrac{\delta_2}{\bar{r}_o}\right)\bar{1}^F\,\bar{1}^H\,\bar{g}}{\left(1-2\bar{\beta}\dfrac{\delta_2}{\bar{r}_o}\right)s\left(s^2+s\bar{\eta}_1+\bar{\eta}^2\right)\left(s^2+\Omega_1{}^2\right)} \end{pmatrix} \tag{35}$$

By isolating the characteristic equation, the flow induced vibro-acoustic natural frequency through the pipe segment can be computed as:

$$\Omega_1{}^2=\left(\frac{3M^2n^2\pi^2}{\delta_1}-\bar{c}_0{}^2\left(\bar{\lambda}_m{}^2+n^2\pi^2\right)\right) \tag{36}$$

By employing Laplace inversion, Eq.(35) in Fourier-Hankel transform plane becomes

$$\bar{P}_0^{FH}\left(\bar{\lambda}_n,\bar{\lambda}_m,\bar{t}\right)=c_0{}^{-2}\left(\left(n\pi(-1)^{n+1}\bar{P}_o^{H}\left(1,\bar{\lambda}_m\right)+n\pi\bar{P}_o^{H}\left(0,\bar{\lambda}_m\right)\right)F_1(\bar{t})+\left(\frac{\left(1-\dfrac{2\bar{h}}{(1-\delta_2)}\dfrac{\delta_2}{\bar{r}_o}\right)\bar{1}^F\,\bar{1}^H\,\bar{g}}{\left(1-2\bar{\beta}\dfrac{\delta_2}{\bar{r}_o}\right)}\right)F_2(\bar{t})\right) \tag{37}$$

where

$$F_1(\bar{t})=\frac{1}{\Omega_1{}^2}\left(1-\cos\Omega_1\bar{t}\right)\quad;\quad F_2(\bar{t})=\begin{pmatrix} \dfrac{1}{\Omega_1{}^2\bar{\eta}^2}+\dfrac{e^{-\beta_1\bar{t}}}{\left(\beta_1{}^2-\bar{\eta}^2\right)\left(\beta_1{}^2+\Omega_1{}^2\right)}+\dfrac{e^{-\beta_2\bar{t}}}{\left(\beta_2{}^2-\bar{\eta}^2\right)\left(\beta_2{}^2+\Omega_1{}^2\right)} \\ -\dfrac{e^{i\Omega_1\bar{t}}}{2\Omega_1{}^2\left(\bar{\eta}^2-\Omega_1{}^2+i\Omega_1\bar{\eta}_1\right)}-\dfrac{e^{-i\Omega_1\bar{t}}}{2\Omega_1{}^2\left(\bar{\eta}^2-\Omega_1{}^2-i\Omega_1\bar{\eta}_1\right)} \end{pmatrix} \tag{38}$$

$$\forall\quad \beta_1=\frac{\bar{\eta}_1}{2}+\left(\frac{\bar{\eta}_1{}^2}{4}-\bar{\eta}^2\right)\quad;\quad \beta_2=\frac{\bar{\eta}_1}{2}-\left(\frac{\bar{\eta}_1{}^2}{4}-\bar{\eta}^2\right)$$

Meanwhile, the Fourier inversion of Eq.(37) takes the form

$$\overline{P}_0^{\,H}\left(\overline{x},\overline{\lambda}_m,\overline{t}\right)=n^4\pi^4\,\overline{1}^{\,H}\,c_0^{\,-2}\begin{pmatrix} 2F_1\left(\overline{t}\right)\left(\overline{P}_o\left(1\right)\sum_{n=1}^{\infty}\left(\frac{\left(-1\right)^{n+1}}{n^3\pi^3}\right)\sin n\pi\overline{x}+\overline{P}_o\left(0\right)\sum_{n=1}^{\infty}\left(\frac{1}{n^3\pi^3}\right)\sin n\pi\overline{x}\right) \\ +\,2F_2\left(\overline{t}\right)\left(\frac{\left(1-\frac{2\overline{h}}{\left(1-\delta_2\right)}\frac{\delta_2}{\overline{r}_o}\right)\overline{g}}{\left(1-2\overline{\beta}\,\frac{\delta_2}{\overline{r}_o}\right)}\right)\sum_{n=1}^{\infty}\left(\frac{\left(1+\left(-1\right)^{n+1}\right)}{n^5\pi^5}\right)\sin n\pi\overline{x} \end{pmatrix} \qquad (39)$$

In the meantime, we can impose the condition that: $\overline{P}_o\left(0\right)=\overline{P}_o\left(1\right)=\overline{P}_1$ **,** in conjunction

with the following mathematical representations namely

$$\sum_{n=1}^{\infty}\frac{\sin n\overline{x}}{n^3}=\frac{\pi^2\overline{x}}{6}-\frac{\pi\overline{x}^2}{4}+\frac{\overline{x}^3}{12}\,,\quad \forall\; 0<\overline{x}<2 \qquad (40a)$$

$$\sum_{n=1}^{\infty}\frac{\sin n\overline{x}}{n^5}=\frac{\pi^4\overline{x}}{90}-\frac{\pi\overline{x}^3}{36}+\frac{\pi\overline{x}^4}{48}-\frac{\overline{x}^5}{240},\quad \forall\; 0<\overline{x}<2 \qquad (40b)$$

to rewrite Eq.(39) as :

$$\overline{P}_0^{\,H}\left(\overline{x},\overline{\lambda}_m,\overline{t}\right)=n^4\pi^4\,\overline{1}^{\,H}\,\overline{c}_0^{\,-2}\begin{pmatrix} 7F_1\left(\overline{t}\right)\overline{P}_1\left(\frac{\overline{x}}{24}-\frac{\overline{x}^2}{16}+\frac{\overline{x}^3}{48}\right) \\ +\,\frac{31}{32}F_2\left(\overline{t}\right)\left(\frac{\left(1-\frac{2\overline{h}}{\left(1-\delta_2\right)}\frac{\delta_2}{\overline{r}_o}\right)\overline{g}}{\left(1-2\overline{\beta}\,\frac{\delta_2}{\overline{r}_o}\right)}\right)\left(\frac{\overline{x}}{45}-\frac{\overline{x}^3}{18}+\frac{\overline{x}^4}{24}-\frac{\overline{x}^5}{120}\right) \end{pmatrix} \qquad (41)$$

$$\forall\;\overline{1}^{\,H}=\frac{\sqrt{2}}{\overline{\lambda}_m}$$

On invoking the Hankel-inversion, the closed form solution for the flow induced acoustic wave can be computed as

$$\overline{P}\left(\overline{x},\overline{r},\overline{t}\right)=\sum_{m=1}^{\infty}\overline{P}_0^{\,H}\left(\overline{x},\overline{\lambda}_m,\overline{t}\right)\frac{\sqrt{2}J_0\left(\overline{\lambda}_m\overline{r}\right)}{\overline{r}_0\overline{J}_1\left(\overline{\lambda}_m\overline{r}_0\right)} \qquad (42)$$

4.0 Analysis of Results

In this paper, flow induced acoustic wave characteristics in a vibrating subsea pipeline is investigated. The pipeline system is idealized as a network of fluid conveyance elastic beams resting on a moving seabed via recent advances in subsea pipeline vibrations. The acoustic-structure dynamic interaction problem is simplified by assuming fully developed internal fluid flow regime. In the formulated problem, a segment of the pipeline is subjected to the action of fluid and excitation forces as illustrated in Fig.1. For the purpose of design exercise, the vibration characteristic of any pipeline system can be influenced by variation in Mach number and ambient sound velocity, as demonstrated in the following mathematical representations viz;

Transverse natural frequency:

$$\overline{\omega}_{n(1)}{}^{2} = \overline{\eta}^{2} - \frac{\overline{\eta}_{1}{}^{2}}{2}\ ;\ \overline{\omega}_{n(2)}{}^{2} = -\overline{\eta}^{2} \qquad \forall$$

$$\overline{\eta}_{1} = \left(\frac{\left(\overline{C}_{1} + \overline{C}_{D} - \left(\frac{\Delta\overline{p}\overline{A}_{o}(1-\frac{\gamma}{2}) + \overline{p}\overline{A}_{o}\gamma + \varepsilon_{t}\beta_{1}\Delta\overline{\Theta}(1-\frac{\gamma}{2}) + \varepsilon_{t}\beta_{1}\overline{\Theta}\gamma}{M\overline{c}_{0}} \right)\right)}{\left(1 - 2\overline{\beta}\,\frac{\delta_{2}}{\overline{r}_{o}}\right)} \right)$$

$$\overline{\eta}^{2} = \left(\frac{n^{4}\pi^{4} - \left((2\sqrt{\delta_{1}}+1)M^{2}\overline{c}_{0}{}^{2} - \beta_{o} + \overline{p}\overline{A}_{o}(1-\frac{\gamma}{2}) + \varepsilon_{t}\beta_{1}(1-\frac{\gamma}{2})\overline{\Theta} \right) n^{2}\pi^{2} + \overline{K}_{b}}{\left(1 - 2\overline{\beta}\,\frac{\delta_{2}}{\overline{r}_{o}}\right)} \right)$$

Longitudinal natural frequency:

$$\overline{\Omega}_{1}^{2} = \frac{\left(-\left(\overline{C}_{2} + \overline{C}_{D} + \frac{\beta_{1}\gamma}{M\overline{c}_{0}} \right)\right)^{2}}{2} + \left(\left(2\sqrt{\delta_{1}}+1\right) M^{2}\overline{c}_{0}{}^{2} - \beta_{1}\left(1 - \gamma/2\right)\right) n^{2}\pi^{2}\ ; \quad \overline{\Omega}_{2}^{2} = -\left(\left(2\sqrt{\delta_{1}}+1\right) M^{2}\overline{c}_{0}{}^{2} - \beta_{1}\left(1 - \gamma/2\right)\right) n^{2}\pi^{2}$$

Acoustics natural frequency:

$$\overline{\Omega}^{2} = \left(\frac{3M^{2}n^{2}\pi^{2}}{\delta_{1}} - \overline{c}_{0}{}^{2}\left(\overline{\lambda}_{m}{}^{2} + n^{2}\pi^{2}\right)\right)$$

with appropriate definitions of terms as computed in Section 3. To enable us make comparison of the natural frequencies, we shall consider the case for which $\gamma \to 0$ to obtain the reduced versions of the foregoing relations viz

$$\overline{\eta}_{1}* = \frac{\left(\overline{C}_{1} + \overline{C}_{D} - \left(\frac{\Delta\overline{p}\overline{A}_{o} + \varepsilon_{t}\beta_{1}\Delta\overline{\Theta}}{M\overline{c}_{0}} \right)\right)}{\left(1 - 2\overline{\beta}\,\frac{\delta_{2}}{\overline{r}_{o}}\right)}\ ; \quad \overline{\eta}*^{2} = \frac{n^{4}\pi^{4} - \left(\begin{array}{l} (2\sqrt{\delta_{1}}+1)M^{2}\overline{c}_{0}{}^{2} - \beta_{o} + \overline{p}\overline{A}_{o} \\ + \varepsilon_{t}\beta_{1}\overline{\Theta} \end{array} \right) n^{2}\pi^{2} + \overline{K}_{b}}{\left(1 - 2\overline{\beta}\,\frac{\delta_{2}}{\overline{r}_{o}}\right)}$$

$$\overline{\Omega}*_{1}^{2} = \frac{\left(-\left(\overline{C}_{2} + \overline{C}_{D}\right)\right)^{2}}{2} + \left(\left(2\sqrt{\delta_{1}}+1\right)M^{2}\overline{c}_{0}{}^{2} - \beta_{1}\right) n^{2}\pi^{2}\ ; \quad \overline{\Omega}*_{2}^{2} = -\left(\left(2\sqrt{\delta_{1}}+1\right)M^{2}\overline{c}_{0}{}^{2} - \beta_{1}\right) n^{2}\pi^{2}$$

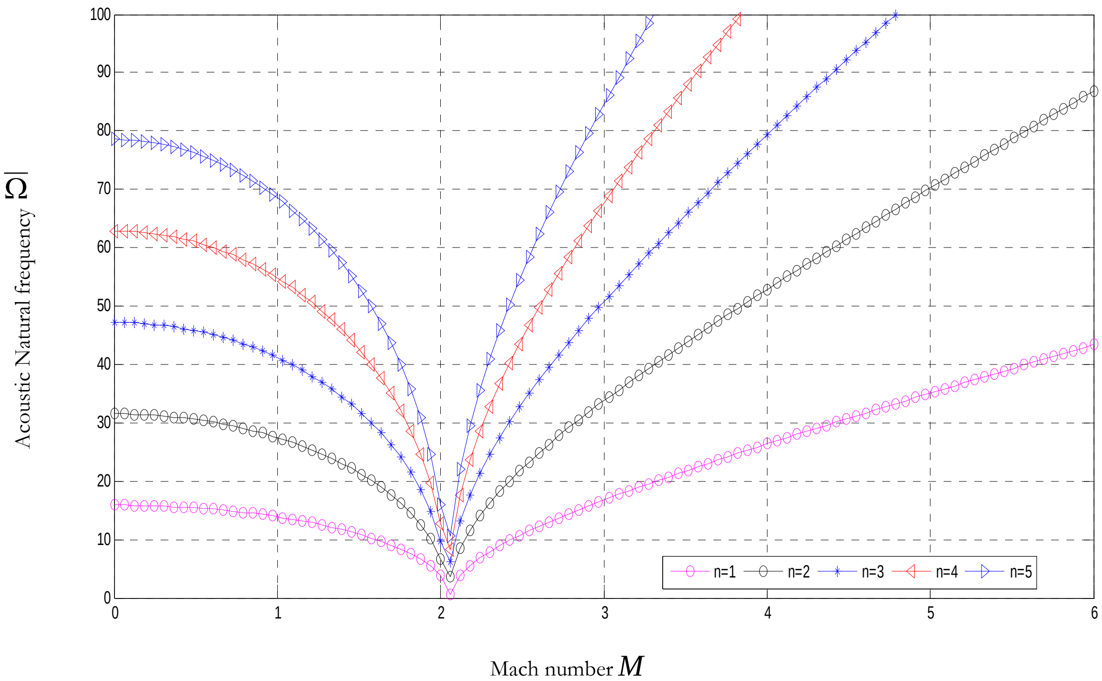

Fig.(2): acoustic natural frequency profile for the case $\delta_1 = 0.5$

With respect to the effect of modal parameters, Fig.2, showed the profiles of acoustic natural frequency for the various modes for a fluid of mass ratio (0.5). As can be seen, the ordering of the family of curves clearly displayed the existence of subsonic/ supersonic and hypersonic zones.

In each zone, the general pattern indicates frequency profiles that are increasing with mode shapes. In the subsonic/supersonic zone, each profile is decreasing monotonically to a critical value. However, beyond the supersonic zone, the natural frequencies are increasing monotonically in the same order.

On the other hand, the effect of fluid mass ratios on the acoustic natural frequency profiles is displayed in Fig.3. In general, the acoustic natural frequency is increasing with increase in mass ratio. As evident from the illustrated picture, the ordering of the curves showed the existence of two zones. In the first zone, the effect of variation in fluid mass ratio is insignificant in the absence of flow.

That is to be expected, as can be seen from the closed form relation derived for the acoustic natural frequency. However, in the neighborhood of sonic flow and beyond, the profiles attenuate to their respective critical values before increasing monotonically in reversed order.

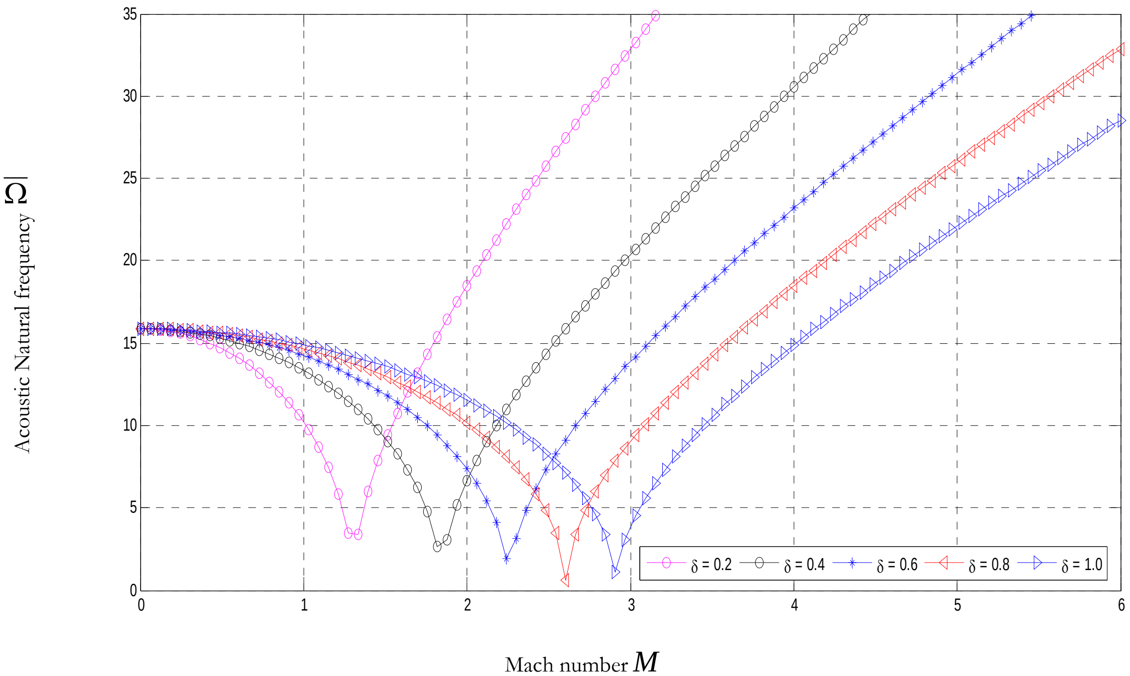

Fig.(3): acoustic natural frequency profile for the case $n = 1$

Comparison of acoustic- and longitudinal- natural frequencies is demonstrated in Fig.4. Nature of profiles indicates that, the acoustics natural frequency is higher in the subsonic zone. However, beyond the subsonic flow, the order is reversed. On the other hand, Fig.5, demonstrates the principal mode profiles for the three natural frequency families associated with flow induced vibration through a fluid conveyance pipe. Although, we note a similar trend, however, the profile of the transverse natural frequency is seen to exhibit an asymptotic behavior in the neighborhood of Mach zero. That is anticipated as evident from the flow velocity kernel couched in the closed form relation for the transverse natural frequency. With respect to the profiles of the acoustic pressure wave, Figs.(6-8) illustrate the pictures of the 3-D plots. In Figs. (6-7), the profiles are essentially parabolic in the direction of flow. In particular, we note in Fig.6, that the acoustic pressure wave oscillates uniformly with time whilst in Fig.7, the pressure wave is attenuating as we approach the wall of the pipe. Such a pattern is expected, since at the pipe wall, the shear stresses attained maximum values. On the other hand, Fig.8 illustrates the pressure profile over cycles of normalized time at the mid section of the pipe segment. As evident from the displayed picture, propagation of acoustic pressure waves is steady with time in the axial direction but attenuates uniformly toward the pipe wall.

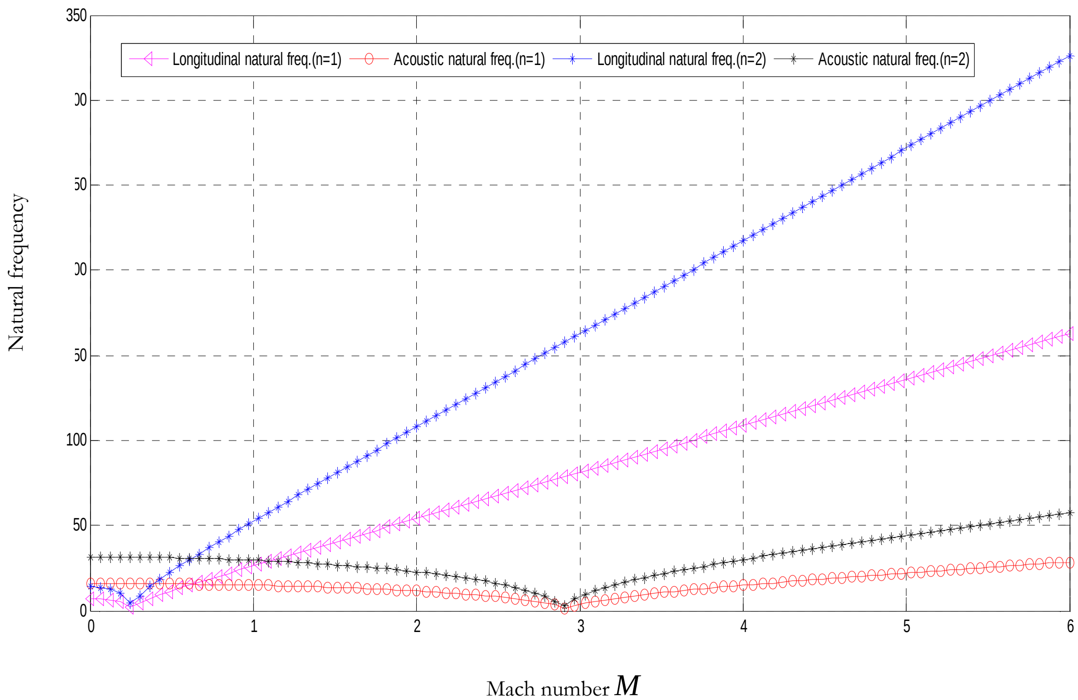

Fig.(4): Acoustic and Longitudinal natural frequency profiles for the case $n=1$

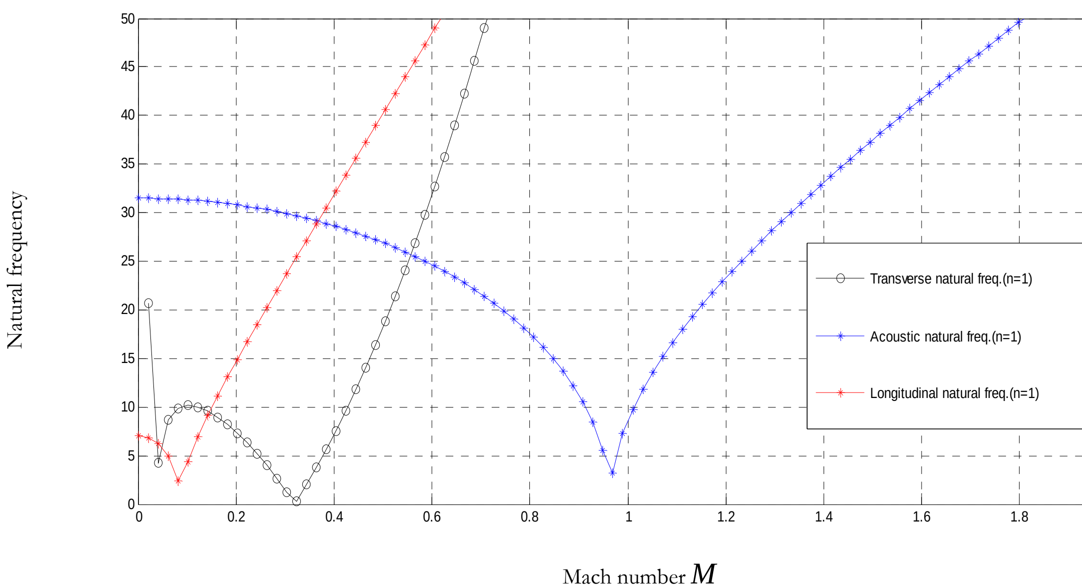

Fig.(5): Comparative natural frequency profiles for the case : $n=1$

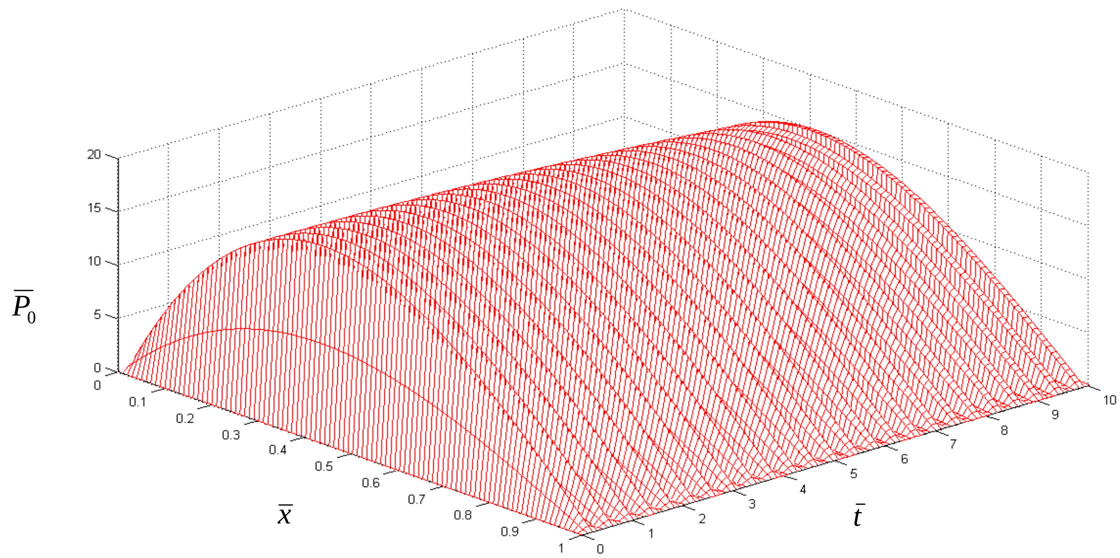

Fig.(6): Acoustic pressure wave profile for the case: $n=1$

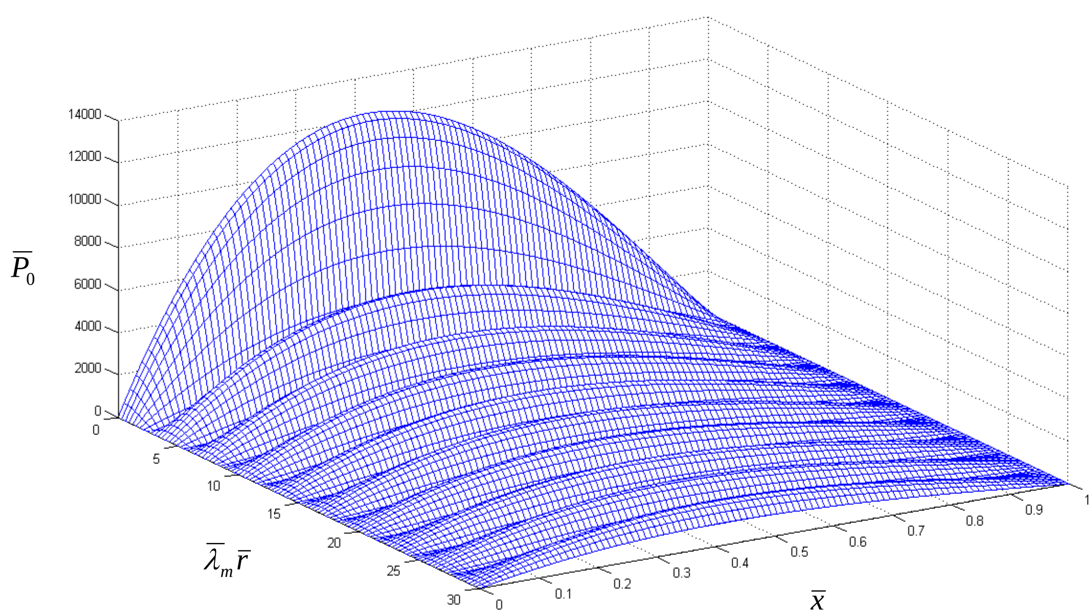

Fig.(7): Acoustic pressure wave profile for the case: $n=1$

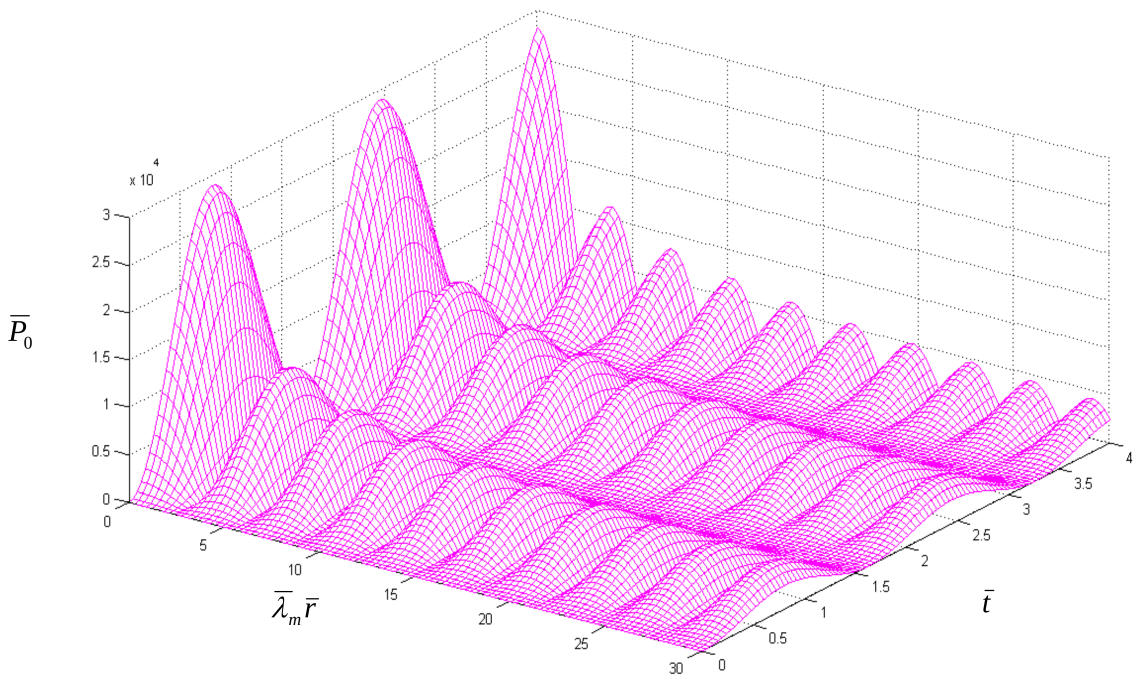

Fig.(7): Acoustic pressure wave profile for the case: $n = 1$

5.0 Conclusion

In this investigation, explicit closed form solutions for the flow induced transverse, longitudinal and vibro-acoustic characteristic frequencies, as well as pressure waves through a pipeline system are derived. The vibro- acoustic properties are shown to be dependent on flow variables and geotechnical properties. The results presented in this study can be positively exploited for field applications.

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Nomenclature

A	pipe cross sectional area after deformation
A_o	original cross sectional area of pipe
A_p	surface area of pipe
A'	change in the surface area of the pipe
C_1	damping force per unit velocity in the transverse direction

C_2	damping force per unit velocity in the axial direction
C_D	hydrodynamic drag coefficient
E	Young modulus of elasticity
$F_1(t)$	external force in the transverse direction
$F_2(t)$	external force in the longitudinal direction
g	acceleration due to gravity
h	depth of pipe below sea level
I	moment of inertia
k_b	stiffness of the sea bed
L	length of pipe
m	sum of the masses of pipe and fluid
m_f	mass of flowing fluid inside the pipe
m_w	mass of sea water displaced by pipe
M	sum of masses of pipe, fluid in pipe and external water displaced by pipe
pA	pressurization effect
P_h	hydrodynamic effect of the ocean
p_o	pressure at entry
T_o	tension in pipe
t	time
u	longitudinal displacement
U	velocity of fluid flowing inside pipe
U'	differential of velocity with respect to x
$\dot{U}$	differential of fluid velocity with respect to time
$\tilde{u}$	longitudinal response in Laplace plane
u^F	longitudinal response in Fourier plane
$\tilde{u}^F$	longitudinal response in Fourier-Laplace plane
w	transverse displacement
$\tilde{w}$	transverse response in Laplace plane
w^F	transverse response in Fourier plane
$\tilde{w}^F$	transverse response in Fourier-Laplace plane
x	axial displacement coordinate
z	transverse displacement coordinate
r_i	internal radius of pipe
r_o	external radius of pipe

Greek letters

α	coefficient of thermal expansivity
γ	coefficient of area deformation
$\Delta\theta$	temperature change from inlet to outlet
Δp	pressure change from inlet to outlet
θ	temperature of the flowing fluid
θ'	temperature gradient
μ	coefficient of sliding friction
∇^2	Laplacian operator
∇	gradient operator
ρ_w	density of water
Φ	velocity potential