A SIMPSON TYPE TRIGONOMETRICALLY FITTED BLOCK SCHEME FOR NUMERICAL INTEGRATION OF OSCILLATORY PROBLEMS

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ABSTRACT

The construction of Second Derivative Trigonometrically Fitted Block Scheme of Simpson Type (TFBSST) of algebraic order 6via collocation techniques is considered for the solution of oscillatory problems. A Continuous Second Derivative Trigonometrically Fitted method (CSDTF) whose coefficients depend on the frequency and step size is constructed using trigonometric basis function. The CSDTF is used to generate the main method and one additional method which are combined and applied in block form as simultaneous numerical integrators. The investigation of the stability properties of the method shows that the method is zero stable, consistent and convergent. Numerical examples presented show that the method is accurate and efficient.

Keywords: Collocation technique; Oscillatory problems; Second Derivative; Simpsons Type; Trigonometrically Fitted methods.

1.0 INTRODUCTION

One of the interesting and important class of initial value problems (IVP) which can arise in practice consists of differential equations whose solutions are known to be periodic or to oscillate with a known frequency. Such problems frequently arise in area such as quantum mechanics, ecology, medical sciences, theoretical chemistry, classical mechanics, theoretical physics and oscillatory motion in a nonlinear force field.

In what follows, we consider the class of first order initial value problem

$$y' = f(x,y), \qquad y(x_0) = y_0 , \qquad x \in [x_0,b] \tag{1}$$

where f satisfies the Lipschitz theorem.

A number of numerical methods based on the use of polynomial function have been proposed for the solution of (1). Akinfenwa *et al.* (2015) proposed a family of continuous third derivative for the integration of oscillatory problems, Adeniran and Ogundare (2015) proposed block hybrid method for the direct integration of second order IVP whose solutions oscillate, Ngwane and Jator

(2012) also proposed hybrid block method for the system of first order IVP involving oscillatory problems while block hybrid Simpson's method was proposed by Biala *et al.* (2015). All these methods were implemented in a block by block fashion. Despite the success of these methods some have low order of accuracy and are expensive in terms of number of function evaluation. Other methods based on exponential fitting techniques have been proposed for the integration of oscillatory problems (see Simos, 1998 and Vanden Berghe *et al.,*1999)). Although these methods are easy to implement but tedious mathematical analysis are involved in obtaining the A-stability.

The numerical integration of oscillatory problems using trigonometric polynomial started in 1961 with Gautschi who proposed the Adams and Störmer type of order p. Neta and Ford (1984) proposed the Nyström and Milne Simpson's type, Neta (1986) constructed families of backward differentiation formula while Sanugi and Evans proposed leap frog method and Runge-Kutta method. All these methods were applied in step by step fashion. Despite the success recorded by these methods, there are some setbacks among which are sensitive to change in the frequency, requirement of the eigenvalues of the Jacobian to be purely imaginary and computational burden.

In the spirit of Gautschi, Psihoyios and Simos (2003 and 2005) proposed trigonometrically fitted schemes for the solution of oscillatory problems which are applied in predictor-corrector mode based on the well-known Adams-Bashforth method as predictor and Adams-Moulton as corrector. The methods are very costly to implement, involve greater human effort and reduced order of accuracy. The use of multistep collocation method for the construction of trigonometrically fitted methods based on trigonometric polynomial have been explored by Jator *et al.* (2013) who proposed Numerov type block methods. Ngwane and Jator (2012, 2013 and 2015) who constructed block hybrid scheme for the integration of oscillatory problems. A family of trigonometrically fitted Enright Second Derivative methods for oscillatory IVP was proposed by Ngwane and Jator (2015).

The current research proposes a block second derivative of Simpson type for the integration of first order and system of first order oscillatory problems via multistep collocation method based on trigonometric polynomial. One incentive for using a basis function other than polynomial is the fact that as every oscillation has to be followed when integrating oscillatory IVP, the rounding error accumulates for small sizes. Methods based on polynomial functions are not

reliable in that case (Duxbury, 1999). According to Lambert (1973) the sum of error over each sub-intervals $[x_0, x_2], [x_2, x_4], ..., [x_{n-2}, x_n]$ for $n > 0$ is the error of the integration over the whole interval $[x_0, x_n]$ if Simpson's rule is used to evaluate $\int_{x_n}^{x_{n+2}} f(x) dx$. Therefore Simpson's rule is an admirable method for quadrature. However, one of the major conditions a numerical method for the solution of IVPs must ensure is that the sub-intervals of the interval of integration must not overlap. In the case of Simpson's method, the sub intervals of integration are $[x_0, x_2], [x_2, x_4], ... [x_{n-2}, x_n]$ which overlap and complicates the accumulation of errors and hence a poor method for integrating IVPs. We therefore show that the block second derivative of Simpson type in this study can be made to overcome this shortcoming and cope with the integration of oscillatory problems.

2.0   METHODOLOGY

In order to obtain CSDTF, the exact solution $y(x)$ is approximated by seeking the solution $y(x, u)$ of the form

$$y(x, u) = \sum_{j=0}^{2k} a_j x^j + a_{2k+1} \sin(\omega x) + a_{2k+2} \cos(\omega x) \qquad (2)$$

Through interpolation of $y(x, u)$ at x_{n+j}, $j = k - 2$, collocation of $\frac{\partial}{\partial x}(y(x, u))$ at the points $x_{n+j}, j = 0(1)k$ and $\frac{\partial^2}{\partial x^2}(y(x, u))$ at the points $x_{n+j}, j = 0(1)k$, we obtain the following $2k + 3$ system of equations.

$$y(x_{n+j}, u) = y_{n+j},$$
$$j = k - 2 \qquad (3)$$

$$\frac{\partial}{\partial x}(y(x_{n+j}, u)) = f_{n+j}, \qquad j = 0(1)k \quad (4)$$

$$\frac{\partial^2}{\partial x^2}(y(x_{n+j}, u)) = g_{n+j}, \qquad j = 0(1)k \quad (5)$$

Equations $(3), (4)$ and (5) are solved to obtain the coefficients of a_j. The values of a_j are substituted into (2) to obtain the CSDTF given by

$$y(x,u)$$
$$= y_{n+k-2} + h \sum_{j=0}^{k} \beta_j(x,u) f_{n+j} + h^2 \sum_{j=0}^{k} \gamma_j(x,u) g_{n+j} \qquad (6)$$

The continuous method (6) is used to generate the main method by evaluating at $x = x_{n+k}$ to obtain

$$y_{n+k} - y_{n+k-2} = h \sum_{j=0}^{k} \beta_j(u) f_{n+j} + h^2 \sum_{j=0}^{k} \gamma_j(u) g_{n+j} \qquad (7)$$

The $k-1$ complementary methodsare obtained by evaluating (6) at $x = x_{n+j} \ j = 1(1)k-1, j \neq k-2$ to obtain

$$y_{n+j} - y_{n+k-2} = h \sum_{\substack{j=0 \\ i=1}}^{k} \overline{\beta_{j,\imath}}(u) f_{n+j} + h^2 \sum_{\substack{j=0 \\ i=1}}^{k} \overline{\gamma_{j,\imath}}(u) g_{n+j} \qquad (8)$$

2.1   Specification of TFBSST
Imposing the conditions given by equations$(2),(3),(4) \text{ and } (5) \text{ for } k = 2$ to obtain system of 7 equations. The system is solved simultaneously for the coefficients $a_j, j = 0(1)6$which are substituted into (2) to obtain the CSDTF as

$$y(x,u)$$
$$= y_n + h \sum_{j=0}^{2} \beta_j(x,u) f_{n+j} + h^2 \sum_{j=0}^{2} \gamma_j(x,u) g_{n+j} \qquad (9)$$

Thus, evaluating (9) at $x = x_{n+2}$ to obtain the main method in the form

$$y_{n+2} - y_n$$
$$= h(\beta_0(u) f_n + \beta_1(u) f_{n+1}$$
$$+ \beta_2(u) f_{n+2})$$
$$+ h^2(\gamma_0(u) g_n + \gamma_1(u) g_{n+1}$$
$$+ \gamma_2(u) g_{n+2}) \qquad (10)$$

with coefficients in both trigonometric form and equivalent power series form given as follows

$$\left. \begin{aligned}
\beta_0 &= \frac{(3u^2+18)\cos(2u)-6u\left(-\frac{1}{6}u^2-1\right)\sin(2u)-24u\left(-\frac{1}{6}u^2-\frac{5}{2}\right)\sin u-12u^2\cos u-27u^2-18}{15u^2\cos(2u)+3u(4u^2\sin(u)+u^2\sin(2u)+4u\cos u-9u+12\sin u-6\sin(2u))} \\[4pt]
\beta_1 &= \frac{(24u^2-36)\cos(2u)-6u\left(-\frac{2}{3}u^2+8\right)\sin(2u)-24u\left(-\frac{2}{3}u^2+2\right)\sin u+48u^2\cos u+36}{15u^2\cos(2u)+3u(4u^2\sin(u)+u^2\sin(2u)+4u\cos u-9u+12\sin u-6\sin(2u))} \\[4pt]
\beta_2 &= \frac{(3u^2+18)\cos(2u)-6u\left(-\frac{1}{6}u^2-1\right)\sin(2u)-24u\left(-\frac{1}{6}u^2-\frac{5}{2}\right)\sin u-12u^2\cos u-27u^2-18}{15u^2\cos(2u)+3u(4u^2\sin(u)+u^2\sin(2u)+4u\cos u-9u+12\sin u-6\sin(2u))} \\[4pt]
\gamma_0 &= \frac{(-u^2+9)\cos(2u)+6u\sin(2u)+24u\sin u-8u^2\cos u-9u^2-9}{15u^2\cos(2u)+3u(4u^2\sin(u)+u^2\sin(2u)+4u\cos u-9u+12\sin u-6\sin(2u))} \\[4pt]
\gamma_1 &= \frac{(u^2-9)\cos(2u)-6u\sin(2u)-24u\sin u+8u^2\cos u+9u^2+9}{15u^2\cos(2u)+3u(4u^2\sin(u)+u^2\sin(2u)+4u\cos u-9u+12\sin u-6\sin(2u))}
\end{aligned} \right\} \qquad (11)$$

Converting equation (11) to equivalent power series form to obtain

$$\left. \begin{aligned}
\beta_0 &= \frac{7}{15}+\frac{4}{1575}u^2+\frac{1}{15750}u^4+\frac{59}{36382500}u^6+\frac{7043}{170270100000}u^8+\cdots \\[4pt]
\beta_1 &= \frac{16}{15}-\frac{8}{1575}u^2-\frac{1}{785}u^4-\frac{59}{18191250}u^6+\frac{7043}{85135050000}u^8+\cdots \\[4pt]
\beta_2 &= \frac{7}{15}+\frac{4}{1575}u^2+\frac{1}{15750}u^4+\frac{59}{36382500}u^6+\frac{7043}{170270100000}u^8+\cdots \\[4pt]
\gamma_0 &= \frac{1}{15}+\frac{2}{1575}u^2+\frac{1}{31500}u^4+\frac{59}{72765000}u^6+\frac{7043}{340540200000}u^8+\cdots \\[4pt]
\gamma_1 &= -\frac{1}{15}-\frac{2}{1575}u^2-\frac{1}{31500}u^4-\frac{59}{72765000}u^6-\frac{7043}{340540200000}u^8+\cdots
\end{aligned} \right\} \qquad (12)$$

The complementary method is obtained by evaluating (9) at $x=x_{n+1}$

$$\begin{aligned}
y_{n+1}-y_n &= h\left(\overline{\beta_{0,1}}(u)f_n+\overline{\beta_{1,1}}(u)f_{n+1}+\overline{\beta_{2,1}}(u)f_{n+2}\right) \\
&\quad + h^2\left(\overline{\gamma_{0,1}}(u)g_n+\overline{\gamma_{1,1}}(u)g_{n+1}\right. \\
&\quad \left. +\overline{\gamma_{2,1}}(u)g_{n+2}\right)
\end{aligned} \qquad (13)$$

with coefficients both trigonometric form and equivalent power series form given as follows

$$\left. \begin{aligned}
\overline{\beta_{0,1}} &= \frac{\frac{1}{24}\left(-6\left(-\frac{7}{6}u^2-10\right)\sin(2u)+(18u^2+144)\cos(2u)-216u\left(-\frac{23}{108}u^2-\frac{7}{9}\right)\sin u-288\cos u-162u^2+144\right)}{u(-9u+12\sin(u)-6\sin(2u)+4\cos u+5u\cos(2u)+4u^2\sin u+u^2\sin(2u))} \\[4pt]
\overline{\beta_{1,1}} &= \frac{\frac{1}{24}\left(-6\left(-\frac{8}{3}u^2+32\right)\sin(2u)+(96u^2-144)\cos(2u)-216u\left(-\frac{8}{27}u^2+\frac{8}{9}\right)\sin u+192u^2\cos u+144\right)}{u(-9u+12\sin(u)-6\sin(2u)+4\cos u+5u\cos(2u)+4u^2\sin u+u^2\sin(2u))} \\[4pt]
\overline{\beta_{2,1}} &= \frac{\frac{1}{24}\left(-6\left(-\frac{1}{6}u^2+2\right)\sin(2u)+6u^2\cos(2u)-216u\left(-\frac{7}{108}u^2-\frac{13}{9}\right)\sin u+(-96u^2+288)\cos u-54u^2-288\right)}{u(-9u+12\sin(u)-6\sin(2u)+4\cos u+5u\cos(2u)+4u^2\sin u+u^2\sin(2u))} \\[4pt]
\overline{\gamma_{0,1}} &= \frac{\frac{1}{24}\left(42u\sin(2u)+(-7u^2+60)\cos(2u)-216u\left(-\frac{1}{36}u^2-\frac{5}{18}\right)\sin u+(-20u^2-96)\cos u-45u^2+36\right)}{u(-9u+12\sin(u)-6\sin(2u)+4\cos u+5u\cos(2u)+4u^2\sin u+u^2\sin(2u))} \\[4pt]
\overline{\gamma_{1,1}} &= \frac{\frac{1}{24}\left(-6(u^2-18)\sin(2u)+(-42u^2+96)\cos(2u)-216u\sin u+(24u^2-384)\cos u+18u^2+288\right)}{u(-9u+12\sin(u)-6\sin(2u)+4\cos u+5u\cos(2u)+4u^2\sin u+u^2\sin(2u))} \\[4pt]
\overline{\gamma_{2,1}} &= \frac{\frac{1}{24}\left(-6u\sin(2u)+(u^2-12)\cos(2u)-216u\left(-\frac{1}{36}u^2+\frac{11}{18}\right)\sin u+(44u^2-96)\cos u+27u^2+108\right)}{u(-9u+12\sin(u)-6\sin(2u)+4\cos u+5u\cos(2u)+4u^2\sin u+u^2\sin(2u))}
\end{aligned} \right\} \qquad (14)$$

Converting equation (14) to equivalent power series form to obtain

$$\left.\begin{aligned}
\overline{\beta_{0,1}} &= \frac{101}{240} + \frac{29}{14400}u^2 + \frac{29}{7056000}u^4 + \frac{62239}{65197440000}u^6 + \frac{923773}{41074387200000}u^8 + \cdots \\
\overline{\beta_{1,1}} &= \frac{8}{15} - \frac{4}{1575}u^2 - \frac{1}{15750}u^4 - \frac{59}{36382500}u^6 - \frac{7043}{170270100000}u^8 + \cdots \\
\overline{\beta_{2,1}} &= \frac{11}{240} + \frac{53}{100800}u^2 + \frac{149}{7056000}u^4 + \frac{43489}{65197440000}u^6 + \frac{10077799}{533967033600000}u^8 + \cdots \\
\overline{\gamma_{0,1}} &= \frac{13}{240} + \frac{89}{100800}u^2 + \frac{137}{7056000}u^4 + \frac{2687}{5927040000}u^6 + \frac{5843587}{533967033600000}u^8 + \cdots \\
\overline{\gamma_{1,1}} &= -\frac{1}{6} + \frac{1}{1008}u^2 + \frac{1}{70560}u^4 + \frac{5}{26078976}u^6 + \frac{103}{42717362688}u^8 + \cdots \\
\overline{\gamma_{2,1}} &= -\frac{1}{80} - \frac{13}{33600}u^2 - \frac{29}{2352000}u^4 - \frac{7769}{21732480000}u^6 - \frac{1733279}{177989011200000}u^8 + \cdots
\end{aligned}\right\} \quad (15)$$

Equations (10) and (13) are the discrete methods whose converted coefficients in power series are given by equations (12) and (15) and are combined to form a block method called the TFBSST.

3.0   RESULTS AND DISCUSSION

3.1   Local Truncation Error of TFBSST
Following Lambert (1973), the local truncation errors of (10) and (13) are better obtained using their series expansion. Thus Local Truncation Error (LTE) of (10) and (13) are respectively as obtained.

$$LTE = \begin{bmatrix} \dfrac{h^7}{9450}\left(y^{(7)}(x_n) + \omega^2 y^{(5)}(x_n)\right) + O(h^8) \\ \dfrac{h^7}{4725}\left(y^{(7)}(x_n) + \omega^2 y^{(5)}(x_n)\right) + O(h^8) \end{bmatrix} \qquad (16)$$

Following the definition of Lambert (1973) and Fatunla (1988), TFBSST is consistent if its order is greater than one. We therefore remark that TFBSST is of algebraic order 6 and hence it is consistent.

3.2   Stability of TFBSST
Following Akinfenwa *et al.* (2015), the TFBSST can be rearranged and rewritten as a matrix difference equation of the form

$$A^{(1)}Y_{w+1} = A^{(0)}Y_w + hB^{(1)}F_{w+1} + hB^{(0)}F_w + h^2 D^{(1)}G_{w+1}$$
$$+ h^2 D^{(0)}G_w \qquad (17)$$

where

$Y_{w+1} = (y_{n+1}, y_{n+2})^T, Y_w = (y_{n-1}, y_n)^T, F_{w+1} = (f_{n+1}, f_{n+2})^T, F_w = (f_{n-1}, f_n)^T,$
$G_{w+1} = (g_{n+1}, g_{n+2})^T, G_w = (g_{n-1}, g_n)^T$ and $A^{(0)}, A^{(1)}, B^{(0)}, B^{(1)}, D^{(0)}, D^{(1)}$ are 2×2 matrices

specified as follows

$$A^{(1)} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, A^{(0)} = \begin{bmatrix} 0 & -1 \\ 0 & -1 \end{bmatrix}, B^{(1)} = \begin{bmatrix} \overline{\beta_1} & \overline{\beta_2} \\ \beta_1 & \beta_2 \end{bmatrix}, B^{(0)} = \begin{bmatrix} 0 & \overline{\beta_0} \\ 0 & \beta_0 \end{bmatrix}, \quad D^{(1)} = \begin{bmatrix} \overline{\gamma_1} & \overline{\gamma_2} \\ \gamma_1 & \gamma_2 \end{bmatrix},$$

$$D^{(0)} = \begin{bmatrix} 0 & \overline{\gamma_0} \\ 0 & \gamma_0 \end{bmatrix}$$

3.3 Zero Stability

According to Lambert (1973) and Fatunla (1988),TFBSST is zero stable if the roots of the first characteristic polynomial have modulus less than or equal to one and those of modulus one are simple. i.e. $\rho(R) = \det[RA^{(1)} - A^{(0)}] = 0$ and $|R_i| \leq 1$. Hence TFBSST is zero stable since from our calculation $|R| = (0,1)$.Since consistence+zero stability=convergence and TFBSST is of order 6 and also zero stable, then it converges in the spirit of Lambert (1973) and Fatunla (1988).

3.4   Linear Stability and Region of Absolute Stability of TFBSST

Applying the block method to the test equations $y' = \lambda y$ and $y'' = \lambda^2 y$ and letting $z = \lambda h$ yields $Y_{w+1} = \xi(z)Y_w$, where $\xi(z) = \frac{A^{(0)} + zB^{(0)} + z^2 D^{(0)}}{A^{(1)} - zB^{(1)} - z^2 D^{(1)}}$. The matrix $\xi(z)$ for TFBSST has eigenvalues given by $(\varphi_1, \varphi_2) = (0, \varphi_2)$, where $\varphi_2 = \frac{\eta_2(z,u)}{\tau_2(z,u)}$ is called the stability function and

$$
\begin{aligned}
\eta_2(z,u) = {} & \left((z^4 - 3z^3 - 31z^2 - 36z + 60) u^2 - 23z^4 - 60z^3 + 12z^2 + 144z \right) u \cos(u)^2 \\
& + \left(\left((-z^3 - 5z^2 - 4z + 12) u^4 + (-8z^4 - 12z^3 + 60z^2 + 120z - 72) u^2 + 24z^2 (z^2 \right.\right. \\
& + 3z + 3) \big) \sin(u) + 2 \left((z^4 - 20z^2 - 36z + 12) u^2 - 4z^2 (z^2 + 3z + 3) \right) u \big) \cos(u) \\
& + \left((-2z^3 - 10z^2 - 8z + 24) u^4 + (-10z^4 - 24z^3 + 48z^2 + 168z + 72) u^2 \right. \\
& - 24z^2 (z^2 + 3z + 3) \big) \sin(u) + 3 \left(\left(z^3 - \frac{1}{3}z^2 - 12z - 28 \right) u^2 + \frac{31}{3}z^4 + 28z^3 \right. \\
& + 4z^2 - 48z \big) u
\end{aligned}
$$

$$\tau_2(z, u) = \left(\left(z^4 + 3z^3 + 11z^2 - 60z - 60\right)u^2 - 23z^4 + 60z^3 - 84z^2\right)u\cos(u)^2 + \left(\left((z \right.\right.$$
$$+ 1)\left(z^2 - 12\right)u^4 + \left(-8z^4 + 12z^3 - 48z^2 + 72z + 72\right)u^2 + 24z^2\left(z^2 - 3z + 3\right)\right)$$
$$\sin(u) + 2\left(\left(z^4 - 26z^2 - 12z - 12\right)u^2 - 4z^2\left(z^2 - 3z - 21\right)\right)u\right)\cos(u) + \left(\left(2z^3\right.\right.$$
$$- 10z^2 - 24z - 24\right)u^4 + \left(-10z^4 + 24z^3 + 156z^2 - 72z - 72\right)u^2 - 24z^2\left(z^2 - 3z\right.$$
$$\left.+ 3\right)\right)\sin(u) - 3\left(\left(z^3 + \frac{31}{3}z^2 - 28z - 28\right)u^2 - \frac{31}{3}z^4 + 28z^3 + 28z^2\right)u$$

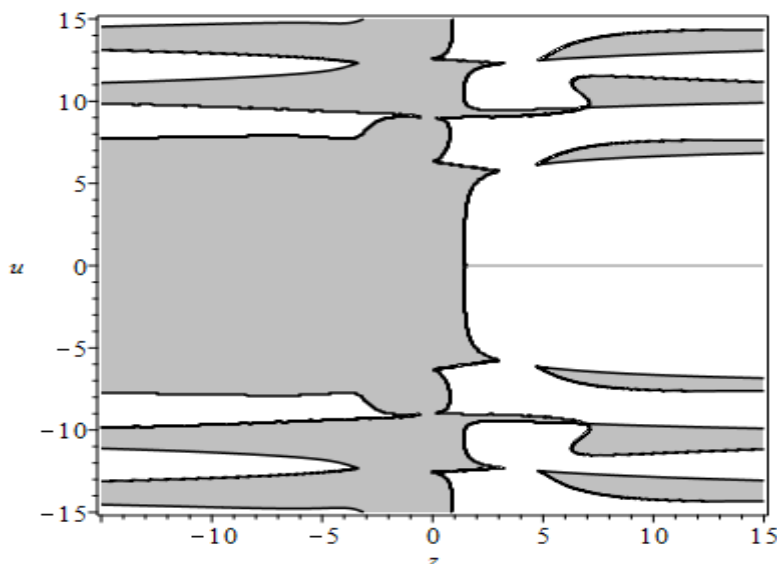The stability region of TFBSST is plotted in Figure 1

Figure 1: Stability plot of TFBSST

4.0   Numerical Examples

In this section, the performance and accuracy of the newly constructed TFBSST for on variety of well-known oscillatory IVPs both linear and nonlinear problems is discussed. The fitted frequency of each problem is used as default frequency for the computation. The absolute errors or maximum error of the approximation solutions are computed and compared with results from existing methods in the literature. An error of the form $r \times 10^{-s}$ is written as $r(-s)$. All computations were carried out using written codes in Maple 2016.1 and executed on Windows 10 operating system. The second order initial value problems among the examples considered were first reduced to their equivalent system of first order initial value problems.

Problem 1: Nonlinear Oscillatory Problem

Consider the nonlinear initial value problem given by

$$y'' = -y(1 + \alpha y^2) + \alpha \cos^3 x, \quad y(0) = 1, \quad y'(0) = 0, \quad \alpha = 0.01 \quad (3.34)$$

This equation is a particular case of the undamped Duffing equation, with a forcing term selected so that the analytical solution is $y(x) = \cos x$. This problem is solved with the newly developed TFBSST with $\omega = 1$ and $h = \frac{\pi}{2^i}, i \geq 2$ in the interval $0 \leq x \leq \frac{33\pi}{4}$. The numerical computation results are compared with the mixed collocation methods IIb of order four, IIIa of order four, IIIb of order six and IVb of order four, all in Duxbury (1999). The comparison of the maximum errors are as presented in table 8.

Table 1: Comparison of Maximum errors

Methods	h				
	$\dfrac{\pi}{4}$	$\dfrac{\pi}{8}$	$\dfrac{\pi}{16}$	$\dfrac{\pi}{32}$	$\dfrac{\pi}{64}$
TFBSST	3.62(−16)	2.55(−20)	1.00(−25)	1.05(−28)	5.10(−29)
IIb	5.78(−15)	2.46(−14)	1.10(−13)	1.34(−13)	1.42(−12)
IIIa	9.44(−15)	7.69(−14)	3.04(−13)	1.44(−12)	4.71(−12)
IIIb	3.55(−15)	9.94(−14)	7.88(−13)	3.64(−13)	6.00(−12)
IVb	1.87(−13)	4.90(−14)	2.21(−13)	2.12(−12)	3.98(−12)

It is clear in table 1 that TFBSST show superiority in terms of accuracy. Hence TFBSST is a better scheme when compared to the existing methods in the literature.

Problem 2 (Mohd Nasir et al. 2015)

Consider the linear initial value problem given by

$$y' = -100(y - \sin x) + \cos x, \quad y(0) = 0$$

whose solution in closed form is given as $y(x) = \sin x$. This problem is solved with the newly developed TFBSST with $\omega = 1$ and $h = \frac{1}{10^i}, i \geq 0$ in the interval $0 \leq x \leq 1$. The numerical computation results are compared with the order four and order five block backward differentiation formula, BBDF(4) and BBDF(5) of Ismail (2010) respectively.

Table 2: Comparison of Maximum errors

h	Max Err of TFBSST	Max Err of BBDF(4)	Max Err of BBDF(5)
1	5.48(−18)	−	−
0.1	7.00(−30)	−	−
0.01	6.00(−30)	8.28814(+006)	5.18981(+013)
0.001	1.60(−29)	1.5545(−003)	1.67200(−005)
0.0001	1.78(−28)	−	−

It is seen from Table 2 that TFBSST performs better than those of Ismail (2010).This clearly demonstrated that the method developed in this paper has almost the same result as the exact as error is as low as 10^{-18} which is infinitesimal for as large step size as $h = 1$.

5.0   CONCLUSION

In this paper, a Second Derivative Trigonometrically Fitted Block Scheme of Simpson Type (TFBSST) of algebraic order 6 was constructed and implemented. The convergence and accuracy of the methods were established The methods was tested with some standard oscillatory problems and was found to be accurate and favorably compare with other methods in the literature as shown in the Tables 1-2 above. In our future research we will consider other specification of TFBSST and discuss the concept of frequency estimation for problems with multifrequencies.

Conflict of Interest: The authors declare that they have no conflict of interest.

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