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DEDICATION

This Thesis is solely dedicated to my parents who supported me enough to reach this pinnacle of education.

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IVb	Extended Mixed Collocation Methods of Algebraic order 4 with 2 stages
2PBOSM	Two-point block one-step method
ARKC	Absolute stable Runge-Kutta Collocation method
BBDF1	Block Backward Differentiation Formula of order four
BBDF2	Block Backward Differentiation Formula of order five
BBDF4	Block Backward Differentiation Formula of order four
BBDF5	Block Backward Differentiation Formula of order five
BDF	Backward Differentiation Formula
BHMTB	Block Hybrid Method with Trigonometric Basis
BHSM2	Block Hybrid Simpson’s Method with Two off-grid points
BHT	Block Hybrid Trigonometrically-Fitted Method
BHTFM	Block Hybrid Method with Trigonometrically-Fitted Method
BTFEBDM	Block Trigonometrically-Fitted Extended Backward Differentiation Method
CHEB I	Zeros of Chebyshev polynomial of the first kind
CHEB II	Zeros of Chebyshev polynomial of the second kind
CHEBY24	Dissipative Chebyshev Exponential-Fitted Methods
CTDBM	Continuous Third Derivative Block methods
DIRK	Diagonally Implicit Runge-Kutta method
DIRKN	Diagonally Implicit Runge-Kutta Nystrom method
E2PBN	Explicit 2-Point 1-Block Method

E3PBN	Explicit 3-Point 1-Block Method
EF	Exponentially Fitted
EFRK	Exponentially Fitted Runge-Kutta
EOPM	Explicit One-step P-Stable Method
EQPTS	Equidistance points in $[x_0, b]$
ERK4	Explicit phase fitted Runge Kutta of order four
ESDM	Enright Second Derivative Method
ETSHM5TF	Explicit Two Step Hybrid Modified Fifth order Trigonometrically-Fitted
ETSHM6TF	Explicit Two Step Hybrid Modified Sixth order Trigonometrically-Fitted
ETSHM7TF	Explicit Two Step Hybrid Modified Seventh order Trigonometrically-Fitted
HLMM	Hybrid Linear Multistep Method of order seven
HMB	Exponentially Fitted Hybrid Method
HSDM	Hybrid Second Derivative Methods
IVP	Initial Value Problem
LMM	Linear Multistep Methods
LTE	Local Truncation Error
MRK	Modified Runge-Kutta-Nyström
N4	Fourth order Runge-Kutta-Nyström Method
NTDRK	New Trigonometrically-Fitted Two-derivative Runge-Kutta
NFE	Number of function evaluation
NVSBBDF	New Variable Step size Block Backward Differentiation Formula
ODE	Ordinary Differential Equation
PDE	Partial Differential Equation
PSM	Power Series Method

RK	Runge-Kutta
RKN	Runge-Kutta-Nyström
SDBBDF	Second Derivative Block Backward Differentiation Formula
SDTFF	Second Derivative Trigonometrically-Fitted Block Backward Differentiation Formula of Adams Type
SSDM	Simpson Second Derivative Method
TBNM	Trigonometrically-Fitted Block Numerov Type Method
TFBBDF2	Second Derivative Trigonometrically-Fitted Block Backward Differentiation Formula for $k = 2$
TFBBDF3	Second Derivative Trigonometrically-Fitted Block Backward Differentiation Formula for $k = 3$
TFBBDF4	Second Derivative Trigonometrically-Fitted Block Backward Differentiation Formula for $k = 4$
TFBSST2	Second Derivative Trigonometrically-Fitted Block Scheme of Simpson's Type for $k = 2$
TFBSST3	Second Derivative Trigonometrically-Fitted Block Scheme of Simpson's Type for $k = 3$
TFBSST4	Second Derivative Trigonometrically-Fitted Block Scheme of Simpson's Type for $k = 4$
TFBTDM	Trigonometrically-Fitted Block Third Derivative Method
TIRK3	Three stages Trigonometrically Implicit Runge-Kutta
TS	Total Step
TSDM	Trigonometrically-Fitted Second Derivative Method
TTRKNM	Trigonometrically-Fitted implicit Third Derivative Runge-Kutta-Nystrom Method
VSSBBDF	Variable Stepsize Superclass Block Backward Differentiation Formula

ABSTRACT

This study develops two classes of Second Derivative Trigonometrically-Fitted Block Schemes for the numerical integration of oscillatory IVPs using collocation techniques. The two classes of methods are the Second Derivative Trigonometrically-Fitted Block Backward Differentiation Formula (TFBBDF) and Second Derivative Trigonometrically-Fitted Block Scheme of Simpson Type (TFBSST). The Trigonometrically-Fitted Methods for each scheme depend on the step size and frequency that are constructed using trigonometric basis function. The continuous Second Derivative Trigonometrically-Fitted Method for each scheme is used to generate the main method. The additional $k-1$ complementary methods for TFBBDF are obtained from the second differentiation of its continuous form, while the complementary methods of TFBSST are obtained from the same continuous method as main method. The main and complimentary methods in their converted power series form are combined and applied in block form as simultaneous numerical integrators. The stability properties for both classes are investigated using boundary locus plot. It is found that both classes are zero stable, consistent and convergent. The class of k`-step TFBBDF is of order $2k+1$ while that of $k$`-step TFBSST of order $2k+2$. Both classes of the methods are applied on a number of numerical examples and the results showed that they are more accurate and more efficient for oscillatory problems when compared with existing methods in the literature reviewed in this work.

Keywords: Backward Differentiation Formula, Collocation technique, Oscillatory Problems, Simpsons Type, Trigonometrically-Fitted methods.